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An Encoding-Complex View of Cognitive Number Processing: Comment on McCloskey, Sokol, and Goodman (1986)

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McCloskey, Sokol, and Goodman (1986) presented a model of verbal-number production that was based on the Arabic-number-reading errors of several brain-damaged subjects. The model assumed that number processing entails separate comprehension, calculation, and production mechanisms interconnected by a single type of abstract quantity code. We propose instead that numbers activate multiple specific representations functionally integrated in an encoding complex. Further analyses of the number-word confusion matrix produced by one of their subjects, HY, showed that his Arabic-number-reading errors were predicted by the visual similarity of digits and by numerical relations (numerical proximity and odd–even agreement). These findings and other research on number processing suggest a more complex encoding architecture than McCloskey et al. assumed and raise questions about their conclusion that HY's deficit is localized within a verbal-number production system, and also about the psychological distinctions among number comprehension, calculation, and production.

This comment examines several assumptions that underlie the McCloskey, Sokol, and Goodman (1986, hereafter M,S,&G) model of verbal-number production, one component of a more general theory of number processing described by McCloskey, Caramazza, and Basili (1985). The theory posits a central calculation system that processes only abstract number representations, and distinct number input (comprehension) and output (production) systems that translate between the unitary abstract format and specific number codes (i.e., Arabic and verbal numbers). Evidence for the theory comes largely from performance dissociations and error patterns observed in several brain-damaged subjects. Here, we focus on their patient HY, a right-handed male who suffered left hemisphere damage at the age of 66. We describe M,S,&G's model and how it accounts for several aspects of HY's performance, and then report further analyses of HY's number-reading errors that suggest an alternative encoding-complex view of number processing. The encoding-complex model proposes an integrated network of format-specific number codes and processes that collectively mediate number comprehension, calculation, and production, without the assumption of central abstract representations. The complex number-encoding structures suggested by our analyses and by other research on number processing imply that the functional architecture of number processing is more difficult to identify than M,S,&G assumed.

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McCloskey, Sokol, and Goodman's Model and HY's Deficit

According to M,S,&G and McCloskey et al. (1985), cognitive number processing involves separate systems that correspond to three distinct cognitive functions: *comprehension*, *calculation*, and *production*. Figure 1 presents a schematic representation of the model adapted from McCloskey et al. (1985, pp. 173, 174, 180). When a number is presented, the comprehension system encodes each element (e.g., digit) in the stimulus into a format-independent (i.e., abstract) semantic quantity code that is the same for verbal and Arabic numbers (M,S,&G, pp. 308, 323). These semantic codes are the basis for subsequent processing in M,S,&G's model (p. 308). For example, the abstract semantic codes constitute the input to the hypothesized calculation system, which stores associative number facts and performs arithmetic computations.

The semantic codes generated by the comprehension or calculation systems also serve as inputs to the verbal or Arabic production subsystems. When naming is required, verbal production processes transform the abstract semantic code into a phonological code for verbal number names. This involves three phases. Syntactic processes first construct a multislot frame for the number, where each slot specifies the appropriate *number-word class* (i.e., "ones," "teens," or "tens" words) and multiplier class (e.g., hundreds, thousands, etc.). Second, the frame is filled with the semantic quantity codes generated by the comprehension or calculation systems. Finally, the number-class specifications in the syntactic frame guide lexical search processes that retrieve number-word representations for the abstract quantity codes. M,S,&G hypothesized that ones (i.e., one, two, ..., nine), teens (ten, eleven, ..., nineteen), and tens (twenty, thirty, ..., ninety) words rep-

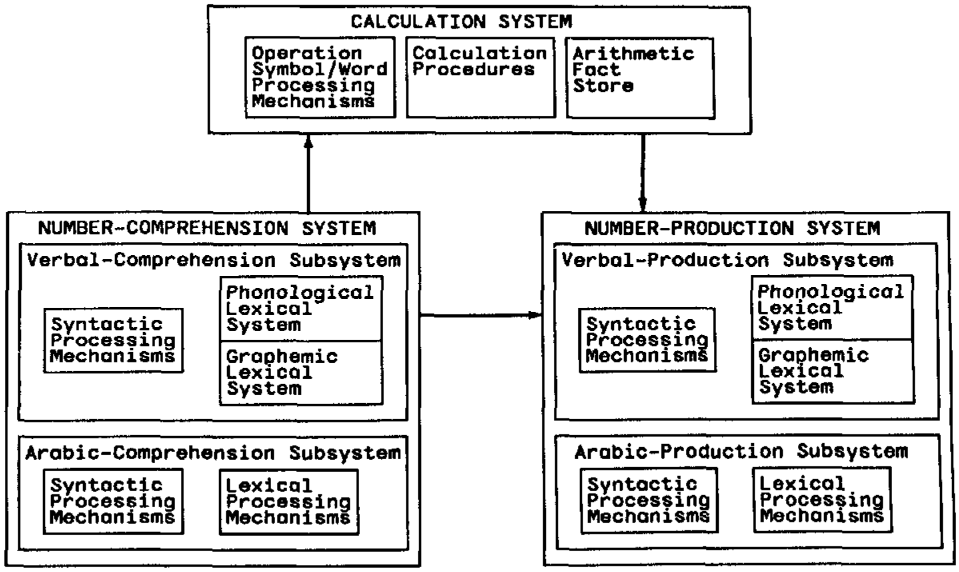

Figure 1. Schematic representation of the number-processing model. (From "Cognitive mechanisms in number processing and calculation: Evidence from dyscalculia" by M. McCloskey, A. Caramazza, and A. Basili, 1985, *Brain and Cognition, 4*, pp. 173, 174, 180. Copyright 1985 by Academic Press. Adapted by permission; arrows represent the transfer of format-independent quantity codes between systems.)

resent distinct frame-addressed lexical subclasses within the production system and are represented as whole-word units (p. 323). That is, for example, *sixteen* is a unit and not a concatenation of *six* plus *teen*.The primary evidence for M,S,&G's model of verbal-number production comes from aphasic subjects who read Arabic numbers aloud. Subjects also performed number-processing tasks that did not require verbal-number production to assess the status of their number comprehension and calculation systems. HY read approximately 5,000 Arabic numbers that varied in length from two to seven digits, with an error rate of 7.1% at the level of individual number words. M,S,&G tallied the frequencies of specific substitution errors for different words (e.g., saying "nineteen" for 17, and "nineteen thousand five hundred ten" for 17,510 represent the same number-word substitution error). The resulting number-word confusion matrix (M,S,&G, p. 312) is reproduced here as Figure 2. One obvious feature of Figure 2 is that there are three clusters of errors, with HY's errors consistently preserving the ones, teens, or tens status of the correct number. That is, HY replaced ones-class words primarily with incorrect words from the ones class, the teens class produced errors from the teens class, and the tens class produced errors from the tens class. Within-class substitution errors accounted for 92% of HY's naming errors. Thus, HY rarely made between-class errors (e.g., saying "three" for *thirteen*).

Although HY showed a substantial impairment in reading numbers, he was accurate on a variety of Arabic-number comprehension and calculation tasks that did not require naming (e.g., 99% correct in an arithmetic-verification task, 100% correct for magnitude comparisons; M,S,&G, pp. 310–311). According to M,S,&G's model, HY's success on these tasks demonstrated that the stimulus digits were being correctly translated into quantity codes by the proposed Arabic-comprehension system, and that his calculation system was also largely unimpaired. Therefore, given the model described earlier, HY's Arabic-number-reading errors must have arisen in the production system; that is, in the translation from abstract quantity to verbal name codes. Moreover, the distinct ones, teens, and tens error clusters were taken as evidence that the production lexicon was organized according to these classes, and that syntactic processing was intact. M,S,&G (p. 316) concluded that HY's impairment was localized in the production process that retrieves number words within each lexical class.

M,S,&G's model for number processing is plausible in many respects, and accounts for some salient and some subtle behaviors of HY and other acalculic subjects. For example, the hypothesis of syntactic frames provides an elegant explanation for the clustering of HY's errors. Moreover, the model addresses the important and ambitious objective of providing an overall framework for organizing and conceptualizing number-related cognitive phenomena. Despite these positive aspects, however, some central features of the model are not compelling. Specifically, the related assumptions of abstract quantity codes, a format-independent calculation system, and very restricted, format-independent interconnections among encoding, calculation, and production processes appear to be at odds with evidence discussed later which suggests that a network of format-specific codes collectively subserves comprehension, calculation and production functions. We will argue that the complexities inherent in number processing cannot readily be accounted for by distinct, modular systems of comprehension, calculation and production, but can be accommodated by assuming specific, multifunction codes. Several aspects of HY's number production are relevant to these issues.

Our attention was first drawn to these issues by some informal observations which suggested that numerical and visual factors may have contributed to the pattern of results

	zero	one	two	three	four	five	six	seven	eight	nine	ten	eleven	twelve	thirteen	fourteen	fifteen	sixteen	seventeen	eighteen	nineteen	twenty	thirty	forty	fifty	sixty	seventy	eighty	ninety
zero	64																											
one		615	15		1	5		2	3	2	1																	
two		10	649	1	7	1	14	1	2	1	1																	
three			14	618	2	27	8	10	3	1		1										2						
four		1	1	2	653	3	7	7	8	3																		
five		1	2	4	9	646	3	17	6	5	1					2												
six			8	3	7	4	649	1	17	1															1			
seven		7	1	1	12	10	2	624		13																5		
eight		1	3	2	8	9	17	7	607	2																		
nine			1		2	2	4	12	1	615	2																	
ten		1	1					1			181				2		1			1	1							
eleven										1	1	158		1	1	1		2										
twelve							1				1		154	1	4	1	30		1		1							
thirteen												2		172		1	3	7										
fourteen													1		164		2	3	1	1								
fifteen											4	2		1	1	129	3	8										
sixteen													1				151	1	2									
seventeen												1		1	4	1		135	2	2								
eighteen													1				11	1	137	2								
nineteen											1	1			1		4	5	2	150								1
twenty			1				2				1		1								263	4	4	3	9		3	
thirty						1		1	1					2							2	277		8	3	3	3	
forty					3										3						7	2	277	1		5	2	2
fifty						1					4					1					3	6	4	266	5	7		
sixty																	1				7	1	1	2	277		3	
seventy								4										1					7	4	2	263	2	1
eighty																			1		3	1	6	1	10	4	247	3
ninety										1											1		1	5	2	10	3	269

Figure 2. Number-word confusion matrix for subject HY (reproduced from McCloskey, Sokol, & Goodman, 1986, p. 312). (Rows represent the correct words and columns represent the words produced by HY.)

in HY's error matrix. There was a trend for substitutions to preserve the odd–even (i.e., multiples of two) status of the number corresponding to the correct word. The percentages of errors showing odd–even agreement of the ones digit (ones and teens classes) or tens digit (tens class) were 59%, 71%, and 60%, respectively, for the three clusters. Although M,S,&G did not observe an influence of numerical proximity (p. 313), HY's tendency to maintain odd–even status reduced errors involving immediate neighbors within each cluster and, hence, made an influence of numerical proximity difficult to detect. Even so, the corners perpendicular to the diagonal were less populated in each within-class cluster (see Figure 2) suggesting a lower probability for numerically distant substitutions. Other substitutions in HY's error matrix (e.g., four given for seven, eight given for five; see Figure 2) suggested further that number words associated with visually similar digits were more likely to be confused. Moreover, these numerical and visual factors appeared to operate similarly across the three error clusters.

It is not clear how M,S,&G would accommodate such findings within their account of HY's deficit, because the present version of M,S,&G's model is not explicit about the process of retrieval within number classes nor about what similarities, if any, might exit across classes in the patterns of HY's errors. M,S,&G suggested that the three classes might involve similar "address systems" (p. 316), which suggests the possibility of parallel error patterns across the clusters. In contrast, however, they provided examples of nonparallel errors between classes from another subject to support the claim that number-word representations are retrieved from the production lexicon in whole-word units (M,S,&G p. 323). Comparable analyses were not reported for HY. Although the model is silent about the specifics of retrieval within lexical classes, influences of visual and numerical factors in HY's naming errors, nonetheless, would appear to be at odds with M,S,&G's hypothesis of simple, format-independent connections among separate comprehension, calculation, and production systems. Odd–even status and numerical proximity influence performance in *calculation* tasks (e.g., Ashcraft & Stazyk, 1981; Krueger, 1986; Miller, Perlmutter, & Keating, 1984; Stazyk, Ashcraft, & Hamann, 1982), but it is not clear within M,S,&G's framework that such factors should influence simple number naming. Similarly, it is unclear, given the assumption that HY's digit encoding is intact, how visual similarity of digits could influence the translation of an abstract quantity code into a verbal number representation.

Before examining these arguments and their implications more closely, we report new analyses of HY's error matrix that statistically confirm our informal observations about the effects of visual similarity, odd–even agreement, and numerical proximity.

Analyses of HY's Within-Cluster Error Patterns

To measure the similarity of the error patterns within the three clusters, we first correlated the incidence of errors over corresponding cells, excluding the special case *zero* that occurs normally in number naming only when the digit 0 appears alone (cf. M,S,&G, pp. 308, 317). The ones number "class" included the items *one* through *nine*. Thus, there are $9 \times 8 = 72$ different within-class substitution errors possible for the ones class, and each possible substitution defines an error cell. The teens class included *ten* through *nineteen*, yielding 90 different possible within-class errors, 72 of which correspond to particular cells in the ones class by virtue of common digits (e.g., correct word of *four* and response of "eight" in the ones class is parallel to correct word of *fourteen* and response of "eighteen" in the teens class). The tens class included eight items (*twenty* through *ninety*) so there were 56 different within-class error cells involving tens words, and these corresponded to 56 cells in each of the ones and tens matrices. For the comparisons of the ones and teens error clusters, the 72 parallel cells were correlated, whereas comparisons involving the tens class used the corresponding 56 cells. Error rates were computed as a function of the total number of opportunities for an observation to fall in each cell (i.e., as a function of each row total in Figure 1) and are presented in the Appendix.

The correlations between percentage of error for the ones–teens comparison, the ones–tens comparison, and the teens–tens comparison were .42, .59, and .61, respectively, demonstrating that similar patterns of errors occur within each of the three clusters. This observation contradicts M,S,&G's (p. 323) empirical grounds (i.e., examples of nonparallel errors between classes) for concluding that ones, teens, and tens words occupy functionally distinct whole-word lexical classes in the proposed production system. The correlations are possibly more consistent with a process of assembled phonology (M,S,&G, p. 316), whereby some teens and tens words are generated by a process that retrieves ones words (e.g., retrieve *six* to generate *sixteen* or *sixty*). Even for teens and tens words that cannot be directly assembled from ones-word phonology or morphology (e.g., *eleven*, *twenty*), however, errors correlated .35 ($df = 51$, $p = .01$) with the corresponding substitutions in the ones cluster, indicating a trend for parallel errors across clusters even where a ones-word assembly process could not be used.

The possibility that the ones, teens, and tens classes were influenced by numerical and visual factors, perhaps in similar ways, was evaluated with analysis of variance and multiple regression techniques to analyze percentage of error within classes as a function of numerical nearness, odd–even agreement, visual similarity, and their interactions. There was one extremely deviant error cell in the matrix. HY responded "sixteen" for *twelve* at a rate nearly seven standard deviations above the mean rate for cells in the teens class. This cell and the parallel cells from the ones class ("six" given for *two*) and from the tens class ("sixty" stated for *twenty*) were excluded from the analyses.[1]

Nearness was operationally defined as 9 minus the absolute value of (error number minus correct number) so that large values represented close numbers. Values below the mean were coded "low" and values above coded "high." The digit in the tens place was used to compute nearness for the tens class. Thus, for example, two versus five, twelve versus fifteen, twenty versus fifty, and the converse errors, all received nearness values of $9 - |5 - 2| = 6$. Odd–even agreement was a dichotomous variable with pairs of correct and error words that disagreed on their odd–even status (e.g., *one–two*, *eight–three*) coded low, and pairs that agreed (e.g., *four–eight*, *nine–three*) coded high. For items in the tens class, odd–even agreement was based on the digit in the tens place. The visual similarity of the 72 single-digit pairs was rated on a 7-point scale by eight individuals who received the pairs in different random orders (alpha = .92). As would be expected for visual judgments, the mean ratings for symmetrical pairs (e.g., 3 vs. 9 and 9 vs. 3) were highly correlated, $r(34) = .96$. The visual

[1] The analyses were also performed without exclusions using log-transformed data. There were no substantial differences in the results of the two sets of analyses.

Table 1
Correlations Among Variables in the Analyses of HY's Arabic-Number-Reading Errors

Variable	%ON	%TE	%TN	AG2	NR	VS	NR2
%TE	.46						
%TN	.56	.61					
AG2	.27	.28	.35				
NR	.34	.22	.27	−.19			
VS	.15	.21	.06	−.14	.06		
NR2	.33	.24	.25	−.25	.86	.16	
VS2	.16	.18	.04	−.10	−.06	.83	.04

Note. n = 71 different within-class substitution errors for the ones- and teens-class correlations. n = 55 for the tens-class correlations. %ON, %TE, %TN = percentage of opportunities to state a particular correct word from the ones, teens, or tens classes that involved a particular within-class substitution error. AG2 = odd/even agreement. NR = numerical nearness. VS = mean rated visual similarity. NR2 = dichotomized nearness. VS2 = dichotomized visual similarity. See text or Appendix for details about each variable.

similarity ratings for the pairs of single digits were assigned to the corresponding cells in the other classes (e.g., the rating based on 2 vs. 5 was also used as the visual similarity measure for 12 vs. 15 and 20 vs. 50). Pairs below the overall mean rating were coded low and pairs above coded high.

The Appendix presents the mean visual-similarity ratings for each pair, the percentage of error for each cell in each class, and the values for other variables. The analyses were based on 71 cells for the ones and teens, and 55 cells for the tens class. Table 1 presents the correlation matrix for the variables used and shows that excluding the extreme cell and its counterparts does not appreciably reduce the correlations among the error clusters, and also that the dichotomized variables for nearness and visual similarity preserve much of the variability in the continuous variables.

The mean percentages of error are presented in Figure 3 as a function of class, numerical nearness, odd–even agreement, and visual similarity. One striking feature of the data is that errors were most common when all three factors favored a confusion, and this pattern was similar across the three clusters. Analyses of variance confirmed these observations. Ones and teens contained the same number of cells and were analyzed together with class as a fourth variable. The three-way interaction of nearness, agreement, and visual similarity was significant, $F(1, 63) = 9.36$, $MS_e = .856$, $p = .003$, and none of the interactions involving class were significant at even the .20 level ($MS_e = .680$). A separate analysis showed that the three-way interaction was also significant for the tens class, $F(1, 47) = 4.00$, $MS_e = .573$, $p = .05$. The numerical and visual variables had other robust effects, the specific nature of which can be inferred from the form of the three-way interaction. In the ones–teens analysis, there were significant main effects of agreement, $F(1, 63) = 18.76$, $p < .001$, nearness, $F(1, 63) = 22.79$, $p < .001$, and visual similarity, $F(1, 63) = 7.27$, $p = .009$, as well as an interaction between nearness and visual similarity, $F(1, 63) = 10.31$, $p = .002$. In the tens class, there was no simple effect of visual similarity ($F < 1$), but the main effects of agreement, $F(1, 47) = 7.91$, $p = .007$, and nearness, $F(1, 47) = 6.70$, $p = .01$, were significant,

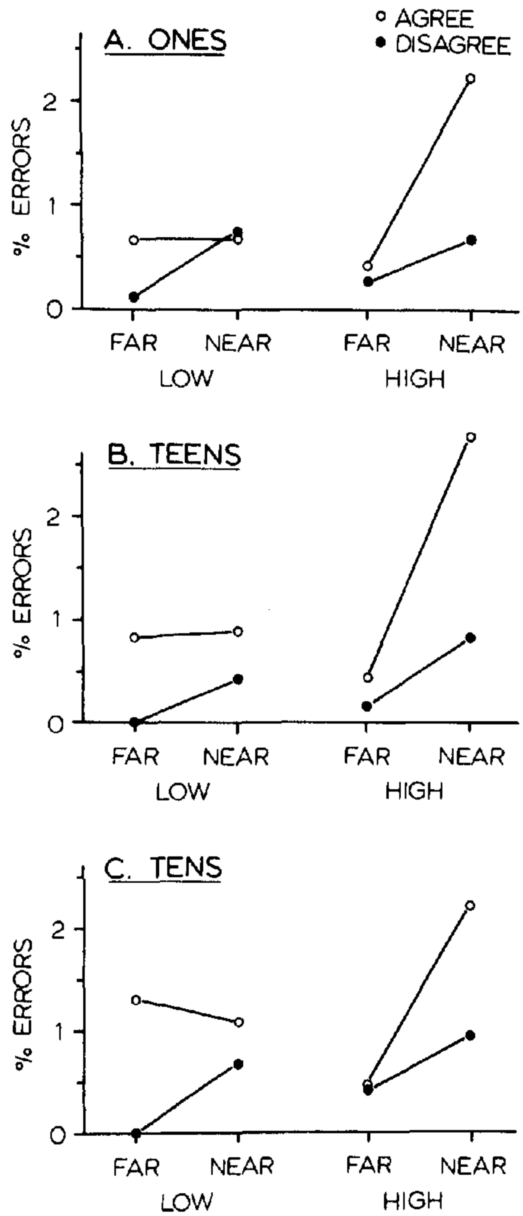

Figure 3. Mean percent errors within HY's ones, teens, and tens error clusters as a function of three kinds of similarity: numerical proximity (far, near), visual similarity (low, high), and odd/even agreement (agree, disagree).

and the interaction between nearness and visual similarity approached significance, $F(1, 47) = 3.05$, $p = .09$.

Regression analyses using nondichotomized (i.e., continuous) values for nearness and visual similarity showed that the three predictors and their interactions accounted for 49% of the variability within the ones error cluster, 32% in the teens, and 34% in the tens. Predicted values for corresponding cells correlated highly (from .83 to .93) and residuals correlated less than the original scores (from .20 to .45). The residual correlations still reached or approached the .05 level of significance (one tailed), suggesting that the three predictors did not capture all the systematic variance shared by the three error clusters.

To estimate the relative contributions of the numerical (nearness and odd–even agreement) and visual factors, separate regressions using only the numerical variables were performed. For the ones cluster, the two numerical variables and their interaction accounted for 26% of the variability and thus visual similarity and its interactions uniquely accounted for an additional 23% (49% − 26%). The numerical and visual factors accounted for 18% and 13%, respectively, in the teens class, and 26% and 8% for tens-class substitution errors. When corresponding cells were averaged across clusters and regressed on the three predictors and their interactions, R^2 was .50, with the numerical variables and their interaction accounting for 31% of the variability and visual similarity and its interactions accounting for 19%.

Implications for Theories of Number Processing

The findings suggest a different view of number representation and processing than that presented by M,S,&G as the basis for their model of number production, and also suggest that number processing may be difficult to partition into discrete functions (and dysfunctions).

M,S,&G's Model and an Encoding-Complex Alternative

Visual and numerical confusions in HY's errors are not predicted by M,S,&G's account of his deficit. That account assumes a number production system that receives only simple, format-independent semantic codes from intact comprehension and calculation systems that are separate from production. Visual and numerical similarity effects on naming errors suggest instead that verbal-number production may be influenced both by surface features that are presumably only relevant to comprehension in M,S,&G's model, and by numerical relations that should be restricted to the calculation system.

Before considering how M,S,&G's model might be modified or extended to accommodate these findings, we describe a fundamentally different basis for a theory of number processing, which we call the *encoding-complex hypothesis.* A strong form of the encoding-complex hypothesis assumes that (a) number processing uses only specific representations, such as imaginal (Hayes, 1973), analogue (Restle, 1970), phonological (Healy & Nairne, 1985) and articulatory (Logie & Baddeley, 1987) codes, as well as other distinct visual, graphemic, and lexical codes (Shepard, Kilpatric, & Cunningham, 1975); (b) the various codes are associatively interconnected so that any code potentially has the capacity to activate other representations within the number complex (e.g., Shepard et al., 1975); and (c) different codes can potentially subserve comprehension, production, and such calculation functions as retrieval of numerical facts and relations (e.g., Gonzalez & Kolers, 1982; Kashiwagi, Kashiwagi, & Hasegawa, 1987; Klein & McInnes, in press), although the strength of specific code–function combinations may depend on an individual's idiosyncratic learning history, culture-specific strategies, and other factors, including brain damage (Boller & Grafman, 1983; Kashiwagi et al., 1987).

The observed effects on naming of visual similarity, odd–even agreement, and numerical proximity follow directly from such a model. Digits activate many codes in different representational modes, including codes for digits and number words that are related to the stimulus by visual similarity, arithmetic relations, or even arbitrary associations. The interaction of visual and numerical factors indicates that the different sources of activation converge on verbal number names, and suggests a model of cascading processes like that proposed by McClellend (1979) for reading. That is, a visual feature analysis activates hypotheses about the identity of each digit, with higher order information related to the candidates (e.g., phonological codes, semantic categories and relations, specific associations) also becoming available. The resulting encoding complex includes various phonological candidates whose strength depends on the amount of priming from visual, semantic, associative, and procedural connections among the multiple representations activated.

An encoding-complex model is consistent with the present results and also with other findings on number processing in normals and aphasics. Several researchers have proposed a link between specific number codes and calculational relations (e.g., counting based on articulatory or lexical associations, Seron & Deloche, 1987; Logie & Baddeley, 1987; see also Boller & Grafman, 1983) and that calculation processes differ for different stimulus formats (Gonzalez & Kolers, 1982, 1987; Klein & McInnes, in press). Specific verbal and visual routes for calculation have also been implicated in the loss and recovery of basic arithmetic skills in aphasic patients (Kashiwagi et al., 1987). In addition to this evidence for code-specific calculation processes, there is evidence that the same factors that mediate HY's number word substitutions also mediate performance in comprehension and calculation tasks. Similarity judgments of numbers, which have been used as a basic measure of semantic number representation (i.e., the product of the comprehension stage in M,S,&G's model), show effects of numerical proximity, odd–even status, and other computational relations (Miller & Gelman, 1983; Shepard et al., 1975); these same factors, which predict HY's verbal production errors, also affect performance in addition, multiplication, and magnitude-comparison tasks (e.g., Ashcraft & Stazyk, 1981; Banks, Fujii, & Kayra-Stuart, 1976; Campbell & Graham, 1985; Krueger, 1986; Miller et al., 1984; Stazyk et al., 1982). Moreover, the distinct clusters found for ratings of conceptual and visual similarity of digits (Shepard et al., 1975, p. 96) are consistent with the separable effects of numerical and visual relatedness on HY's errors. The encoding-complex view is also supported by much research indicating that multiple numerical associations or procedures are activated in parallel and produce interference in tasks involving either nonverbal (LeFevre, Bisanz, & Mrkonjic, 1988; Stazyk et al., 1982; Zbrodoff & Logan, 1986) or verbal-number responses (Campbell, 1987a, 1987b; Campbell &

Graham, 1985). Taken together, these findings support the encoding-complex theory that number comprehension, calculation, and production are coextensive functions subserved by activation over a shared network of format-specific codes, rather than distinct modular systems interconnected by a unitary type of abstract representation.

According to an encoding-complex theory, HY's naming errors were promoted when several different sources of stimulus-related information converged on a competing response (see Figure 3), making that response difficult to inhibit. Similar interference models have been proposed for other aphasic dysfunctions, such as picture naming (e.g., Mills, Knox, Juola, & Salmon, 1979; Rochford & Williams, 1962a, 1962b). Mills et al., for example, found that aphasics and nonaphasics differed more at naming high-uncertainty pictures with multiple alternative names than at naming pictures with single names. HY was even more impaired at naming pictures (M,S,&G, p. 328) than numbers, perhaps because pictures evoke more or stronger competing names than numbers. The encoding-complex view is also consistent with theories that explain normal picture naming and related processing in terms of multiple, specific, and interconnected codes (e.g., Clark & Paivio, 1987; Paivio, 1986), without appeals to a completely abstract level of representation (cf. Clark, 1987).

Finally, note that the encoding-complex model is compatible with those aspects of the MSG model of number production that were central to their explanation of HY's error clusters. The notion of frames, for example, could still be used even if the entries in the frames were multiple, specific number codes. The encoding-complex hypothesis naturally raises questions about whether the frames and associated syntactic processes are identical for the various number representations, but this does not, in principle, conflict with M,S,&G's approach to syntactic processes for numbers.

M,S,&G's model might, of course, be modified or extended to accommodate the effects of visual and numerical factors on naming. For example, the effects of numerical factors on HY's errors might be explained by proposing that even in a simple naming task, there is automatic input from the calculation system to the production system. It might also be hypothesized that counting by ones or twos creates associations between verbal number representations within the production lexicon, and that some direct connections exist between the digit comprehension system and the verbal production system. These associations could account for the observed numerical and visual confusions. Such modifications, however, weaken the strong modularity that distinguishes M,S,&G's approach from the encoding-complex approach. Moreover, modifications that weaken the modularity of the systems may compromise some of M,S,&G's conclusions about the localization of numerical functions and dysfunctions.

Implications for Localization

M,S,&G and McCloskey et al. (1985) used simple task dissociations to localize number processing dysfunctions in HY and other patients. Although not immediately apparent, the conclusion that HY's deficit is localized in production depends to a large extent on the assumption of single abstract codes that define the functional boundaries between comprehension, calculation, and production processes. The logical and empirical bases for such localization become less certain, however, if comprehension, calculation, and production functions are mediated by shared format-specific representations and processes. Such possibilities arise from an encoding-complex view, or from a modified version of M,S,&G's model that permits within-system and between-system connections to bypass the hypothesized abstract codes.

Given these more complex architectural assumptions, successful performance of one task given a particular input format (e.g., digits) does not logically eliminate a specific comprehension (i.e., encoding) deficit as the source of impairment on a different task involving that input type. Rather, the effects of an encoding deficit may be task specific, only emerging in interaction with other task-specific processes. For example, HY's largely intact ability to transform number words to written digits (M,S,&G, p. 310) could demonstrate only the integrity of encoding for that specific task and not necessarily indicate an intact general capacity for number words to access the semantic or associative components required for other tasks. Indeed, although HY's number-word comprehension appeared to be intact in several tasks (e.g., number-word to digit transcoding, Arabic-verbal matching, M,S,&G, p. 310), and despite perfect performance on magnitude comparisons given digits, HY nonetheless had difficulty comparing magnitudes given number words (McCloskey et al., 1985, p. 178). HY's impaired magnitude comparisons for number words cannot clearly be localized in either comprehension or calculation, because the impairment appeared only with a specific combination of input format and number task. This observation appears to contradict M,S,&G's model, which predicts that magnitude comparisons on number words should be essentially perfect when both number-word comprehension and magnitude comparisons via digits are intact. This apparent dissociation is consistent with the encoding-complex hypothesis that calculation processes can be format specific.

Localizing deficits in production as opposed to encoding or calculation is similarly problematic under an encoding-complex view. Consider the observation that HY performed comprehension and calculation tasks more poorly with verbal output than with written output (M,S,&G, pp. 310–311). M,S,&G's assumption that both verbal and written response modes depend on a common code from comprehension and calculation processes leads to the inference that verbal production is dysfunctional: Unimpaired performance via written output is assumed by M,S,&G to confirm the integrity of the number-encoding processes required for naming. In an encoding-complex view, however, verbal-production mechanisms are activated by a variety of specific codes, as HY's errors suggest, and the representations that subserve written as opposed to spoken output may be subject to a different weighting of these factors or even to a different set of factors. Consequently, abnormal "up stream" processing of specific encoding components may selectively impair output via one response mode, even when both verbal and written response mechanisms are intact.

The encoding-complex approach accounts for format- and task-specific performance dissociations that appear to be problematic for M,S,&G's model, such as the pattern of acalculic dysfunction and recovery observed by Kashiwagi et al. (1987). They studied Japanese aphasics with impaired verbal production of multiplication facts, which most Japanese learn as verbal rhymes. Although the patients largely failed to relearn the multiplication facts with verbal presentation and responses, the facts could be relearned given visual presentation combined with written responses. Without systematic information on other number competencies and dysfunctions shown by these subjects, only tentative inferences from these findings are possible. Nevertheless, such dissociations are suggestive of format-specific calculation processes with selective connections to different response modes.

Under an encoding-complex hypothesis, then, the ability to perform nonnaming tasks provides only weak evidence that errors appearing selectively on a naming task are due to a localized production deficit. Although it is possible that HY has an impairment localized in lexical retrieval, once it is allowed that number production processes are mediated by more complex encoding structures than the simple quantity codes assumed by M,S,&G, then the data are also consistent with specific encoding dysfunctions that are more directly tied to verbal responding than to written responding. An encoding-complex view also raises the possibility that even if a deficit could be localized in production, a systematic pattern of naming errors may arise from a different site. For example, given an impairment in lexical retrieval, HY's specific confusions could still originate in the structure of associations among other codes that activate lexical representations.

Another aspect of the localization issue concerns the extent to which a specific representation or process can be defined as uniquely subserving only comprehension, calculation, or production functions. Given an encoding-complex view, these terms denote task demands that result in specific activation, retrieval, and comparison processes operating within an integrated, multicode representational structure. There is no simple way to carve the underlying representational system into distinct comprehension, calculation, and production pieces. For example, specific verbal codes (e.g., phonological, articulatory) that presumably support production functions also appear to function as representational media for calculational associations (Healy & Nairne, 1985; Kashiwagi et al., 1987; Logie & Baddeley, 1987), which in turn appear to be central to measures of number comprehension (e.g., Shepard et al., 1975). Because phonological and other representations potentially subserve comprehension, calculation, and production functions, it is difficult to view these functions as distinct systems or to unambiguously localize dysfunctions in terms of such systems. A deficit in phonological processing that disrupts production of names, could also disrupt performance on comprehension or calculation tasks that rely on information stored as associations among phonological codes, although the encoding-complex view permits alternative processing strategies that could bypass the damage.

Although encoding numbers, retrieving relational number information, and generating number names must involve some distinct mechanisms, the evidence suggests a more interactive and integrated approach to these functions than M,S,&G's model assumed. Under an encoding-complex view, general terms such as comprehension, calculation, and production are unlikely to map onto underlying representations in a simple one-to-one fashion, as they do in M,S,&G's and McCloskey et al.'s (1985) model. Sharp boundaries between these functions do not exist because numbers activate an ensemble of specific representations, any of which may potentially contribute to comprehension, calculation, or production in certain tasks. Under these assumptions, the dissociations observed by M,S,&G and by McCloskey et al. (1985) may not correspond to a natural taxonomy of cognitive systems, but rather may depend on interactions between specific task requirements and a network of multiple, specific codes that can subserve many functions.

Concluding Comments

We agree with M,S,&G (p. 326) about the value of single case studies of such individuals as HY, but the encoding-complex hypothesis suggests that many aspects of number processing can be realized in different ways as a function of cultural (e.g., Kashiwagi et al., 1987) or idiosyncratic experiences. This complicates interpretation of individual cases. It would be possible to draw stronger inferences from HY's task dissociations and from the other subjects tested by M,S&G and McCloskey et al. (1985) given experimental designs that completely crossed different types of number input (e.g., digits and words) with different types of output (e.g., digital, verbal, nonverbal) for a variety of tasks at the same point in time. M,S,&G used an impressive array of tasks, but they did not report results from HY for all input–output by task combinations. The questions we have raised about task-specific encoding structures and about the distinctions among comprehension, calculation, and production processes suggest that complete experimental designs of this sort are crucial for determining whether deficits are format specific or format independent, and for determining finally how useful it will be to view number processing as a small set of distinct, modular systems.

Even given more balanced experimental designs, the interactions among the processes underlying encoding, calculation, and production make it necessary to analyze performance in a variety of tasks in exacting detail. In particular, comparisons of specific patterns of errors *across* tasks may help to identify with greater certainty the cognitive components that these tasks share, and hence to associate performance deficits with these components. This will require that detailed models of individual tasks be developed, as recommended by M,S,&G, and that the assumptions that drive the theoretical interpretations of different tasks be scrutinized and empirically evaluated. Despite our belief that the cognitive number processing system is more complicated than M,S,&G assumed, research such as that of M,S,&G which incorporates research strategies and theoretical approaches from neuropsychology and traditional cognitive psychology, has great potential to contribute to our understanding, not only of dysfunctional and functional number processing, but also of human cognition in general.

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Appendix

Table A1
Variables Used for the Analyses of HY's Within-Class Errors

A	N	V	CW	Rs	%Ones	%Teens	%Tens	Nr	Vis
1	1	1	1	6	.000	.000	—	4	1.13
1	1	1	1	8	.466	.000	—	2	1.00
1	1	1	2	9	.146	.000	.000	2	2.88
1	1	1	6	1	.000	.000	—	4	1.25
1	1	1	8	1	.152	.000	—	2	1.00
1	1	2	2	7	.146	.000	.000	4	3.63
1	1	2	3	8	.437	.000	.997	4	6.00
1	1	2	4	9	.438	.581	.662	4	3.75
1	1	2	7	2	.148	.000	.000	4	3.50
1	1	2	8	3	.305	.000	.362	4	5.63
1	1	2	9	2	.156	.000	.342	2	3.25
1	1	2	9	4	.313	.606	.342	4	3.25
1	2	1	1	2	2.329	.000	—	8	1.25
1	2	1	1	4	.155	.606	—	6	2.75
1	2	1	2	1	1.456	.000	—	8	1.13
1	2	1	3	4	.292	.000	.000	8	1.75
1	2	1	3	6	1.166	1.622	.997	6	3.00
1	2	1	4	1	.146	.000	—	6	2.50
1	2	1	4	3	.292	.000	.662	8	1.75
1	2	1	4	5	.438	.000	.331	8	2.00
1	2	1	5	4	1.293	.676	1.347	8	2.25
1	2	1	6	7	.145	.645	.000	8	1.75
1	2	1	7	6	.296	.000	.704	8	1.88
1	2	1	7	8	.000	1.370	7.704	8	2.50
1	2	1	8	7	1.067	.658	1.449	8	2.50
1	2	2	2	3	.146	.515	1.375	8	3.38
1	2	2	2	5	.146	.515	1.031	6	4.00
1	2	2	3	2	2.041	.000	.664	8	3.38
1	2	2	4	7	1.022	1.744	1.656	6	3.50
1	2	2	5	2	.287	.000	1.010	6	4.50
1	2	2	5	6	.431	2.027	1.684	8	4.38
1	2	2	5	8	.862	.000	.000	6	4.13
1	2	2	6	3	.434	.000	.342	6	3.50
1	2	2	6	5	.579	.000	.685	8	3.75
1	2	2	6	9	.145	.000	.000	6	6.50
1	2	2	7	4	1.778	2.740	2.465	6	3.75
1	2	2	8	5	1.372	.000	.362	6	4.13
1	2	2	8	9	.305	1.316	1.087	8	5.75
1	2	2	9	6	.626	2.424	.685	6	6.25
1	2	2	9	8	.156	1.212	1.027	8	5.63
2	1	1	1	5	.776	.606	—	5	1.13
2	1	1	1	9	.311	.000	—	1	1.63
2	1	1	2	6	2.038	15.464	3.093	5	2.38
2	1	1	3	7	1.458	3.784	.997	5	1.88
2	1	1	4	8	1.168	.581	.662	5	1.75
2	1	1	5	1	.144	1.351	—	5	1.25
2	1	1	6	2	1.158	.645	2.397	5	2.50
2	1	1	7	3	.148	.685	.000	5	2.00
2	1	1	8	4	1.220	.000	2.174	5	2.13
2	1	1	9	1	.000	.606	—	1	1.63
2	1	1	9	5	.313	.000	1.712	5	2.75
2	1	2	1	7	.311	1.212	—	3	3.13
2	1	2	2	8	.291	.515	1.031	3	3.88
2	1	2	3	9	.146	.000	.000	3	4.00
2	1	2	5	9	.718	.000	.000	5	3.38
2	1	2	7	1	1.037	.685	—	3	3.50
2	1	2	8	2	.457	.658	1.087	3	3.38
2	1	2	9	3	.000	.000	.000	3	4.88
2	2	1	1	3	.000	.606	—	7	1.00
2	2	1	2	4	1.019	2.062	1.375	7	1.88
2	2	1	3	1	.000	1.081	—	7	1.00

(Appendix continued)

Table A1 *(continued)*

A	N	V	CW	Rs	%Ones	%Teens	%Tens	Nr	Vis
2	2	1	4	2	.146	.581	2.318	7	1.88
2	2	1	4	6	1.022	1.163	.000	7	2.25
2	2	1	6	4	1.013	.000	.342	7	2.13
2	2	1	7	5	1.481	.685	1.408	7	2.25
2	2	2	3	5	3.936	.541	2.658	7	4.13
2	2	2	5	3	.575	.676	2.020	7	4.13
2	2	2	5	7	2.443	5.405	2.357	7	3.50
2	2	2	6	8	2.460	1.290	1.027	7	5.88
2	2	2	7	9	1.926	1.370	.352	7	3.88
2	2	2	8	6	2.591	7.237	3.623	7	5.13
2	2	2	9	7	1.878	3.030	3.425	7	4.25

Note. CW = correct word (e.g., 2 = *two*, *twelve*, and *twenty* in the ones, teens, and tens class, respectively). Rs = incorrect within-class word stated by HY. %Ones, %Teens, %Tens = percentage of opportunities to state CW that involved Rs. Nr = numerical nearness = (9 − |CW − Rs|). Vis = mean rated visual similarity of CW and Rs (single-digit pairs rated on 1 [low] to 7 [high] scale). Mean Nr = 5.667. Mean Vis = 3.059. A = odd-even agreement of CW and Rs (1 = disagree, 2 = agree). N = dichotomous nearness (1 = far [> mean Nr], 2 = near [< mean Nr]). V = dichotomous visual similarity (1 = low [< mean Vis], 2 = high [> mean Vis]).

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