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Integrated versus Modular Theories of Number Skills and Acalculia

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This paper contrasts two views of the cognitive architecture underlying numerical skills and acalculia. According to the *abstract-modular* theory (e.g., McCloskey, Caramazza, & Basili, 1985), number processing is comprised of independent comprehension, calculation, and production subsystems that communicate via a single type of abstract quantity code. The alternative, *specific-integrated* theory (e.g., Campbell & Clark, 1988), proposes that visuospatial, verbal, and other modality-specific number codes are associatively connected as an encoding complex and that different facets of number processing generally involve common, rather than independent, processes. The hypothesis of specific number codes is supported by conceptual inadequacies of abstract codes, format-specific phenomena in calculation, the diversity of acalculias and individual differences in number processing, lateralization issues, and the role of format-specific codes in working memory. The integrated, associative view of number processing is supported by the dependence of modular views on abstract codes and other conceptual inadequacies, evidence for integrated associative networks in calculation tasks, acalculia phenomena, shortcomings in modular architectures for number-processing dissociations, close ties between semantic and verbal aspects of numbers, and continuities between number and nonnumber processing. These numerous logical and empirical considerations challenge the abstract-modular theory and support the en-

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coding-complex view that number processing is effected by integrated associative networks of modality-specific number codes.

The *encoding-complex* theory of number processing (Campbell & Clark, 1988) assumes that number concepts are represented by multiple format- and modality-specific codes interconnected in a complex network. Excitatory and inhibitory associative mechanisms determine the specific patterns of codes activated on any given task. These basic assumptions about specific number representations and integrated associative processes can explain normal and dysfunctional performance on diverse number-processing tasks and have implications for the putative independence of different numerical processes (i.e, comprehension, calculation, and production), the specificity of dissociations in dysfunctional number processing, and other aspects of acalculia.

The encoding-complex hypothesis is offered, in part, as an antithesis to abstract-modular theories of number processing that currently overshadow alternative views about number processing. Abstract-modular theories assume that number meaning is based on a single type of abstract quantity code and that a small number of separate cognitive "modules" are specialized for distinct aspects of number processing. McCloskey, Caramazza, and their colleagues have developed one well-articulated model of this class that has been described extensively elsewhere (e.g., Caramazza & McCloskey, 1987; McCloskey & Caramazza, 1987; McCloskey, Caramazza, & Basili, 1985; McCloskey, Sokol, & Goodman, 1986; Sokol, Goodman-Schulman, & McCloskey, 1989). The major differences between the two views of number processing concern the abstractness of number codes and the modularity of number processing. The remainder of this paper is organized around these two issues, which are introduced briefly here.

With respect to the concreteness of number codes, McCloskey et al. (1985; Sokol et al., 1989) claim that different surface forms of numbers (e.g., digits and number words) are translated into or derived from a single, modality-independent abstract code. This abstract code underlies arithmetic and other numerical operations and permits translation from one surface form to another. In contrast, the encoding-complex view assumes that calculation and related processes are based on modality-specific number codes that can be directly interconnected without mediation by an abstract code. Specific verbal codes include articulatory, auditory, orthographic, and motor codes for various spoken and written number words, as well as unique codes for special populations (e.g., sign language). The nonverbal codes involved in number processing include visual and motor codes for digits, imaginal and other analogue codes for magnitude (e.g., number lines), and combined visual–motor representation for various number-related activities (e.g., finger counting, using an abacus). Because they are associatively connected, specific codes can activate one another to produce a multicomponent pattern of activation that we call an encoding complex.

The modularity of number processing is a closely linked issue that also distinguishes the two views. McCloskey et al. (1985; Sokol et al., 1989) proposed three distinct number-processing subsystems that correspond to number "comprehension," "calculation," and "production" modules. For example, comprehension is defined as the activation of abstract number codes, and it is the specialized function of the comprehension module to translate surface forms of numbers into abstract number representations. The encoding-complex theory rejects this strong form of modularity and instead asserts that overlapping processes and shared underlying representations and associative mechanisms contribute to comprehension, calculation, and production tasks, although these may differ somewhat in terms of specific information and response requirements. We propose that number comprehension involves the activation of a rich and varied collection of mental representations, rather than the simple activation of a single discrete abstract code, and that calculation can be based on associations between specific number codes, which McCloskey et al. restrict to comprehension and production modules. Furthermore, the basic associative and related mechanisms that underlie number processing are not dedicated exclusively to numerical tasks and therefore are not properly characterized by distinct number processing modules (cf. Gardner, 1983).

This paper reviews rational and empirical considerations that favor the specific-integrated (encoding-complex) view of number processing over the abstract-modular view. Throughout, we examine implications of the encoding-complex hypothesis for understanding the diverse number processing dysfunctions that are generically labeled aculculia, identify some strengths of our theoretical approach, and address some alleged weaknesses. We argue that abstract-modular theories oversimplify the cognitive processes that underlie number processing and should be reevaluated. In particular, thoughtful analysis is crucial for the interrelated assumptions of abstract number representations and modular processing because proponents of abstract-modular theories afford a critical position in their basic research and in their functional diagnoses of acalculic disorders to these assumptions. Despite the dominance of abstract-modular theories in current thinking about normal and dysfunctional number processing, we will show that abstract number representations and distinct number-processing modules have limited explanatory capacity and have difficulty accounting for much of the available empirical evidence, including clinical findings on acalculia. Moreover, rejection of overly restrictive assumptions about the abstractness of number codes and the modularity of number processes can lead to powerful, elegant, and mechanistic models for diverse facets of number processing.

SPECIFIC OR ABSTRACT NUMBER REPRESENTATIONS?

Specific number codes are necessary to explain how people use number words, digits, and other quantitative stimuli. The existence of specific number codes is evident in our capacity to think about the same number in different modalities; for example, to imagine "seeing" or "writing" the digit "9" as on a blackboard or to mentally "hear" the sound of the word "nine." Even theories that assume abstract representations for numbers posit modality-specific codes to mediate the encoding of visual and verbal stimuli as abstract codes and the translation of abstract codes into articulatory and manual responses (cf. McCloskey et al., 1985; Sokol et al., 1989). Diverse experimental findings reviewed later also indicate a central role for modality-specific codes in number processing. Given the need for specific codes, a fundamental question is whether the addition of abstract codes is superfluous or furthers our conceptual understanding of the meaning of numbers and our capacity to explain empirical phenomena. Later sections demonstrate the empirical inadequacies of abstract number codes, but such codes are also conceptually wanting in a variety of ways.

Conceptual Inadequacies of Abstract Codes

One conceptual problem with the McCloskey et al. theory is that the hypothesized abstract codes fail to add much theoretical substance beyond that provided by the hypothesis of specific number codes. To illustrate, McCloskey et al. (1986, p. 308) state that "the internal representation of the Arabic number 8,043 or the verbal number *eight thousand forty-three* might take the form {8}10EXP3, {4}10EXP1, {3}10EXP0. The digits in braces stand for semantic representations of the basic quantities in the number and 10EXP*n* indicates the power of ten associated with each quantity." Thus, we are told that the meaning of 8,043 for people can be understood in terms of abstract quantity codes (digits with braces around them, although presumably number words with braces would serve as well) and some power operator (10EXP*n*), which itself includes such unexplained entities as the base number 10, EXP (for exponent), and other numbers (*n*) representing the particular power of each quantity. But positing abstract quantity representations such as {8} does not further our understanding of the mental representation of numbers beyond what is provided by format-specific representations for digits (8) and words (eight), and the 10EXP*n* operator adds little above equivalent representations stated in terms of specific codes (8/eight thousands, 4/four tens, and 3/three ones).

The possible arbitrariness of the "power of ten" or "10EXP*n*" construct is further demonstrated by considering a McCloskey et al. model for representation of numbers in other bases. Machine language computer programmers, for example, often use numbers in base 2 (binary 0 or 1), base 8 (octal 0 to 7), or base 16 (hexadecimal 0 to F, with letters A to F representing values from 10 to 15 in each place). Presumably, numbers in these bases would be represented by abstract number codes based on "8EXP*n*" constructs and the like. But the proliferation of base-specific abstract number codes adds little to standard notational systems that use modality-specific codes and fails to address fundamental psychological issues about how bases, which are themselves numbers, are represented and how numbers are translated from one base to another. It thus appears that positing an EXP operator evades rather than explains the difficult question of the meaning of numbers in different place positions (e.g., 8_{---} or eight-thousand).

A related conceptual problem with the abstract codes proposed by McCloskey et al. is their apparent circularity. The abstract number code is defined as an abstract quantity code plus the power of ten (or other base) associated with its position. But both the base (10 in the standard model) and the power (*n* in the model) are numbers and therefore need to be represented somehow. If the power of ten (*n*) is itself represented by an abstract digit plus its appropriate power of ten, then an infinite regress is started (e.g., {3}10EXP1 would be decomposed as {3}10EXP[{1}10EXP0], and so on). Alternatively, if *n* can be defined without appealing to the notion of power of ten, then presumably other numbers could be likewise represented and need not be represented by an abstract code using the power of ten construct. Representation of the base value is problematic for similar reasons; for example, the 10 in 10EXP*n* is presumably represented either as {1}10EXP1, which is circular, or as some unspecified code that does not depend on the 10EXP*n* operator, rendering the latter superfluous.

In addition to being conceptually circular, the McCloskey et al. abstract codes do not further our neuropsychological understanding of number representations. That is, the proposed abstract number codes do not suggest physiological mechanisms that are realizable given present knowledge about the human nervous system. For example, the notion of "power of ten" cannot be accounted for by any physical mechanism, nor is it at all clear how to build such an operator from known neuropsychological mechanisms. As noted above, the "power of ten" code is constructed from other abstract quantitative concepts that themselves need to be explained. This lack of concrete mechanisms seriously limits the value of abstract codes, especially for theories relevant to the effects of brain insult on number processing.

Even if the conceptual shortcomings of abstract codes could be overcome, and compelling evidence for abstract codes were found, models of number processing that ignore the unique properties of specific codes would still not be justified. Number processes could only be delegated entirely to a single abstract quantity code given direct evidence that specific codes are irrelevant to number meaning and skill. But this assumption is unwarranted, as the following evidence demonstrates.

Format-Specific Phenomena in Number Processing

The most direct evidence for nonabstract number codes comes from research demonstrating that stimulus format (e.g., digits vs. number words) influences performance in tasks requiring calculation or judgments of such numerical attributes as magnitude and odd–even status. Such findings challenge the assumption that, irrespective of presentation format, number processing is carried out entirely on modality-independent, abstract codes. Results that show an interaction between stimulus format and calculation variables are particularly problematic for the abstract-code view, because such interactions cannot be explained easily by simple differences in the comprehension or production of different stimulus modes. That is, the relative ease of comprehending or producing different number formats (e.g., digits and words) explains only main effects of format on errors or reaction times and does not explain interactive effects of format on calculation processes given the McCloskey et al. assumption that all calculation is performed on abstract number codes. Specific number codes are implicated in a variety of findings, including research on numerical magnitude, odd–even status, and basic number facts.

Numerical magnitude. Relative magnitude appears to be a major component of the abstract quantity codes proposed by McCloskey et al. (1985; see Sokol et al., 1989, p. 108). However, research on mental comparisons of number, size, and other quantitative attributes suggests that relative magnitude may be based on visuospatial and other modality-specific representations rather than on a single, abstract code (e.g., Foltz, Poltrock, & Potts, 1984; Paivio, 1975).

The hypothesis that comparisons of numerical magnitude involve spatial, analogue codes is consistent with several phenomena that are observed across numerical, imaginal, and perceptual dimensions. Mental comparisons for numbers are slower for close numbers than for far numbers (e.g., Moyer, 1973), a symbolic distance effect that has also been observed for judgments about the relative size of imagined objects (Kosslyn, 1975; Paivio, 1975) and that has been interpreted in terms of analogue, imaginal codes. Moreover, people are generally faster to choose the smaller of two small numbers or the larger of two large numbers than to choose the larger of two small numbers or the smaller of two large numbers (Banks, Fujii, & Kayra-Stuart, 1976). This congruity effect also occurs for perceptual (Audley & Wallis, 1964; Wallis & Audley, 1964) and other conceptual judgments (e.g., Ellis, 1972; Marschark, 1983). These parallels between numerical and perceptual comparisons suggest that visuospatial codes (e.g., positions on a number line) contribute to number comparisons.

Differential access to such specific codes would produce modality-specific differences between digits and words on number comparison tasks. This prediction has been supported by studies in which subjects choose the numerically larger number when the physical sizes of stimuli are congruent or incongruent with object size (e.g., Besner & Coltheart, 1979; Paivio, 1975). The general finding is that performance is facilitated by size congruency and disrupted by size incongruency. That is, relative to control conditions, subjects are faster to choose the larger number if it is also physically larger than the alternative, and slower to choose the larger number if it is physically smaller than the alternative. Foltz et al. (1984) confirmed that this congruency effect occurs for number stimuli and that it varies with stimulus format. For digits, the negative effect of size incongruence decreased as numerical distance between the digits increased. In contrast, the incongruity effect was constant for words, regardless of numerical distance. This Stroop-like incongruity effect is also larger for Arabic digits and Chinese characters than for visually presented English number words (Tzeng & Wang, 1983), and larger for number pairs written in such idiographic scripts as kanji than for numbers written in such syllabic scripts as kana and hindi (e.g., Hatta, 1983; Takahashi & Green, 1983; Vaid, 1985).

One interpretation of the interaction between stimulus mode and numerical distance is that the relative contribution of visuospatial and verbal codes to magnitude judgments depends on stimulus mode (i.e., word or digit) or such specific features as type of script. Consistent with this interpretation, Vaid and Corina (1989) demonstrated that magnitude judgments vary as a function of stimulus format, prior experience, and the visual field to which stimuli are presented. Using English–American Sign Language (ASL) bilinguals, they compared size-incongruity effects for digits, English words, and ASL number signs. For words and ASL signs, interference was greater when stimuli were presented to the right visual field (RVF; i.e., left hemisphere), whereas interference was greater for digits presented to the left visual field (LVF; see also Vaid, 1985). Vaid and Corina concluded that magnitude judgments about words and digits involve internal representations that are differentially localized in the left and right hemispheres, respectively. Consistent with the conclusion that digits are processed by the right hemisphere, Katz (1980) also reported a left visual field advantage for magnitude comparisons of digits (but see Besner, Grimsell, & Davis, 1979). These various findings confirm that modality-specific number codes play a central role in magnitude comparison.

Odd–even status. Although proponents of the abstract-code hypothesis have stated that "the role of odd–even status in calculation tasks is . . . quite uncertain" (Sokol et al., 1989, p. 108), odd and even are in fact defined and presumably determined by numerical calculations (e.g., multiple or nonmultiple of 2, n MOD 2 equal to 0 or 1), and odd–even relations have been shown to influence performance on diverse numerical tasks. Sum and product verification are influenced by odd–even status (Krueger, 1986; Krueger & Hallford, 1984), and multidimensional scaling of similarity ratings for number pairs indicates that odd–even status is a highly salient component of number meaning, along with numerical magnitude (Miller & Gelman, 1983; Shepard, Kilpatric & Cunningham, 1975).

As with magnitude judgments, there is evidence that processing of odd–even status differs as a function of format. Comparing the speed of odd versus even judgments, Hines (1990) found that, overall, odd judgments were slower than even judgments, and a comparison between two of his studies indicates that the even-number superiority was smaller for digits (4 msec in Experiment 2) than for words (20 msec in Experiment 5). Hines suggested that the observed RT and error differences occurred because odd and even are linguistically unmarked and marked constructs, respectively, and that this property may be more strongly evoked by number words than by digits (p. 46).

Consistent with the view that odd–even status is processed differently for verbal and nonverbal number formats, there is also evidence (again similar to the results for magnitude comparisons) that odd–even judgments are differentially lateralized as a function of format. Klein and McInnes (1988a, b) reported that odd–even judgments of the numbers one through six were about 100 msec (40%) faster for digits than for words, and words were about 50 msec faster than quantities presented as dots. Presentation format interacted with visual field such that odd–even decisions were faster in the RVF for number words, but faster in the LVF for digits and dot stimuli. Although the laterality effects may not reflect differences in the role of phonological codes between numbers and digits, as Klein and McInnes proposed (see Besner & Bryden, 1988), the results nonetheless suggest a left-hemisphere advantage for odd–even judgments for verbal-number stimuli. This conclusion is also supported by EEG recordings made while subjects make odd–even judgments about auditorially presented numbers (Shepherd & Gale, 1982). Thus, there is evidence that odd–even judgments are based on multiple, format-specific codes that may be differently lateralized, rather than a single abstract code presumably stored in some yet unspecified brain region.

Basic number facts. The abstract-code hypothesis implies that the same representations and processes mediate retrieval of number facts irrespective of format (e.g., Sokol et al., 1989, p. 109). This claim is weakened by the fact that calculation performance often differs for different surface forms of numbers (e.g., Marsh & Maki, 1976; Mayer, 1982; McClain & Shih Huang, 1982). Gonzalez and Kolers (1982, 1987), for example, found that format (Arabic vs. Roman numerals) produced different RT patterns in a sum-verification task, even after subjects were given extensive practice identifying both types of numbers. They viewed the results as inconsistent with the assumption that different surface forms are converted to a common code. Other format-specific effects, however, have not always led researchers to conclude in favor of format-specific processes (e.g., Marsh & Maki, 1976).

Campbell (1990b) studied the effects of word versus digit format in detail by alternating the format of simple multiplication and addition problems between successive trials. Both response times and error rates were higher with the word format, despite small differences in identification times and accuracy, and the deficit with the word format was greater for more difficult problems. Errors involving intrusion of the names of multipliers (e.g., four × *eight* = twenty *eight*) were more common for word-format problems, suggesting greater interference from competing verbal responses with word stimuli. Words also were more affected than digits by the number of different problems associated with a given product (the so-called "fan" effect; e.g., Pirolli & Anderson, 1985), which implies that number-word stimuli generally were more susceptible to interference. The assumption of specific number codes permits several explanations for these findings. Campbell proposed that the highly familiar digit format (e.g., 3 × 9 = ?) supported both visual and verbal-auditory routes to products, whereas retrieval of number-word stimuli (e.g., three × nine = ?) was mediated primarily by verbal-auditory codes. Greater prior experience with digit than word problems could also develop modality-specific associations that would reduce interference from related problems, or perhaps interfering reading responses for words disrupted processing of phonologically based multiplication facts. Selective interference effects for digits and words provided additional strong evidence for format-specific calculation processes and are described in the section on modularity (Campbell, 1990a).

Modality-specific calculation processes are also supported by Kashiwagi, Kashiwagi, and Hasegawa's (1987) research on Japanese aphasics who had impaired performance for simple multiplication operations, which the Japanese typically learn as verbal rhymes. Despite extensive practice, the patients could not relearn multiplication with verbal presentation and responses. They did, however, learn to generate the multiplication facts given visual presentation combined with written responses. Such findings demonstrate that retrieval of number facts can involve multiple codes that are differentially involved as a function of format, that brain injury can selectively disrupt calculation performance for a specific format, and that modality-specific training can compensate for lost number-processing abilities. Some of the preceding evidence for format effects on calculation might be explained by comprehension or production processes (e.g., speed of translating different formats to abstract codes), but that remains to be demonstrated and such explanations are strained by various interactions between format and calculation-related variables.

Our discussion of magnitude judgments, odd–even status, and calculation has emphasized differences in number processing as a function of stimulus mode, but similarities also exist between arithmetic performance whether operants are presented as digits or words (e.g., Campbell, 1990b; Sokol et al., 1989). Although sometimes presented as positive evidence for abstract number codes that are completely neutral with respect to perceptual-motor modalities, specific code models include several mechanisms that explain such findings. The associative interconnections among the multiple codes could result in one stimulus format (e.g., words) activating the entire encoding complex, including codes for alternative formats (e.g., digits). Different stimulus codes may all be translated into a dominant number-word or visual-digit format (i.e, common but specific code), analogous to phonological recoding in reading. It is sometimes not recognized that specific code theories can generally mimic abstract codes, although the reverse is not true. Even exemplar theories based on more specific codes than digits and number words (i.e, idiosyncratic instances) can account for apparently abstract processing on the basis of instance similarity. Evidence for abstract codes will need to be much more precise than it is at present to permit rejection of these specific code alternatives.

Neuropsychological Evidence for Specific Codes

The research literature on acalculia, individual differences, brain lateralization, and other factors relevant to the neuropsychology of number processing supports the encoding-complex view that numerical tasks involve a rich collection of specific codes and associated modality-specific processes. The distinction between verbal and visuospatial codes (or more speculatively, left and right hemispheres) plays a central role in organizing this literature and reflects a major dimension of variation among modality-specific codes. The specific and variable number processes demonstrated by the following phenomena are not easily characterized in terms of a single, modality independent abstract code.

The complexity of number skill dysfunctions. A wide range of quite specific dysfunctions are generically labeled acalculia and provide evidence consistent with the encoding-complex view that number processing involves diverse format-specific representations and processes. In general, multiple brain regions contribute to numerical processing (e.g., Gardner, 1983) and injury to different areas of the brain can produce diverse number dysfunctions (e.g., Luria, 1980). In Benton's (1987, p. 113) words, "Acalculia is a multifaceted concept encompassing a variety of impairments having to do with number operations." Although the results are complicated, there is some evidence that insults to localized regions of the brain disrupt facets of number processing associated with verbal and visuospatial processes.

Verbal components of number processing (e.g., phonological codes) should be more disrupted by damage to the left hemisphere, and there is much support for a relation between acalculia and left-hemisphere damage (Boller & Grafman, 1983). For example, left-hemisphere damage disrupts orally presented computations (Jackson & Warrington, 1986) and interferes with number comprehension more than does right-hemisphere damage (Dahmen, Hartje, Bussing, & Sturm, 1982). Benton (1987) notes that almost all anarithmetic patients, who have difficulty performing basic mathematical operations, demonstrate word finding and other aphasic dysfunctions. Although abstract codes might be reconciled with the correlation among left-hemisphere damage, aphasia, and acalculia by assuming that abstract codes for both numbers and nonnumerical concepts are localized in the left hemisphere, it seems equally plausible and less speculative to explain the phenomena in terms of disrupted associative networks of specific number word codes and the well-founded lateralization of verbal codes to the left hemisphere.

According to the encoding-complex view, specific nonverbal codes and associated processes also play a central role in number processing. A visuospatial component to number processing is suggested by the parallels between magnitude comparisons for numbers and perceptual continua, as reviewed earlier; links between individual differences in spatial abilities and number processing (e.g., Burnett, Lane, & Dratt, 1979); and reports of imaginal number forms (Oswald, 1960; Spalding & Zangwill, 1950). Localized damage to visuospatial and associated number codes has been more difficult to demonstrate, perhaps because both hemispheres contribute to such visuospatial processes as imagery (e.g., Kosslyn, 1988) and because "spatial" aspects of acalculia have not been precisely defined (cf. Benton, 1987, p. 118). Nonetheless, some evidence indicates that distinct number-processing dysfunctions are associated with visuospatial processes. Dahmen et al. (1982), for example, demonstrated that patients with right-hemisphere lesions or with temporal–parietal, left-hemisphere lesions showed particular dysfunctions on a number task that implicates visuospatial processes, specifically comprehension of number operation signs. For recent reviews of research on the relation among number dysfunctions, spatial ability, and the right hemisphere, see Hartje (1987) and Semrud-Clikeman and Hynd (1990).

One implication of modality-specific processing would be that one mode of number processing could sometimes be damaged without dramatic effects on peformance. If number facts are represented both visually and verbally in a particular individual, for example, then one system could operate alone without obvious deficits. Such redundancy suggests that brain insults to number processing may be more common than indicated by gross measures and that specialized tests may be necessary to identify the dysfunction (e.g., addition under conditions of visual or auditory interference). Redundant mechanisms also imply that unused modalities can provide new cognitive bases to recover impaired mathematical capacities. As noted previously, Japanese aphasics who lost the capacity to multiply using verbal rhymes relearned multiplication using nonverbal sensory-motor codes and relations (Kashiwagi et al., 1987). Damage or recovery via redundant systems is less consistent with the hypothesis of a single, abstract calculation system.

Individual differences. The diversity of representations and interconnections in the encoding-complex view suggests that arithmetic and other numerical operations can be carried out in diverse ways and that people can acquire culturally unique and even idiosyncratic sets of representations and rules. There is in fact much variability in how different people perform mathematics problems (e.g., Hayes, 1973; Mayer, 1982), with skilled performers and expert calculators using a particularly broad range of procedures, methods, and strategies (Cancelli, Bergan, & Taber, 1980; Hope, 1987; Hope & Sherrill, 1987). Moreover, these different strategies often implicate verbal and visual–spatial processes. Along these lines, Wertheimer (1959) describes how children can arrive at the solution for the area of a parallelogram by learning a verbal rule "Area equals height times length" or by spatial transformation of the presented figure. The methods and representations that individuals use to perform a given task are determined in part by their learning history relevant to that task. Vaid and Corina (1989), for example, showed that the direction of hemispheric asymmetries for number size judgments for words and digits varied with relative skill in English and ASL.

Evidence that modality-related variables contribute to individual differences in number processing is particularly relevant to the present case. Neuropsychological, educational, and psychological findings indeed implicate visuospatial ability in certain aspects of mathematical skill (Burnett et al., 1979; Rourke, 1978; Sherman, 1967), although the relations are not always strong. A relation between numerical and spatial ability has also been implicated in results showing that numerical ability correlates with gender (e.g., Benbow, 1988; Benbow & Stanley, 1980; Connor & Serbin, 1985; Linn & Peterson, 1985), handedness (Annett & Manning, 1990; Benbow & Benbow, 1984), and other variables of neuropsychological interest. Although consistent with our emphasis on modality-specific codes, many of these relations are controversial and the relevance of some findings can be questioned. Most of the evidence for gender differences, for example, comes from reasoning and related quantitative tasks, rather than the basic operations emphasized in this paper, for which gender differences are not generally reported (Annett & Manning, 1990). Nonetheless, some educational research suggests that girls may have special difficulty with even basic aspects of mathematics; for example, equal numbers of girls and boys demonstrate pure arithmetic learning disabilities without language dysfunction (Ozols & Rourke, 1988; Rourke, 1978), despite the fact that boys far outnumber girls for other learning disabilities.

One important source of individual differences in number processing arises from cultural variation in the way in which calculation is taught and the kinds of devices used to perform numerical operations. There is considerable evidence, for example, that abacus or soroban (Japanese abacus) users rely more heavily on imagery representations for numbers than do nonexperts. Hatta, Hirose, Ikeda, and Fukuhara (1989) found that memory for auditorially presented number sequences was disrupted more by interpolated soroban figures than interpolated digit sequences for soroban experts, but the reverse pattern occurred for control subjects. Hishitani (1990) found several effects implicating imagery in the representation of numbers in abacus experts. One striking finding was that RT increased dramatically with the position of the probed digit for nonexperts, but did not vary with probe position for abacus experts over the range of positions that could be represented imaginally.

Neuropsychologists have studied extensively gender, handedness, and other subject variables that have been implicated in modality-specific number processing. Consequently, theories of number processing that include mechanisms (e.g., modality-specific processes) relevant to these individual differences benefit from and can accommodate available knowledge about such factors as gender (e.g., commentary to Benbow, 1988) and learning disabilities (e.g., Hooper & Wallis, 1989). Abstract-code theories that do not directly address modality and related dimensions of individual differences may require many post hoc assumptions to account for such phenomena.

One consequence of variation in number processing as a function of biological and environmental factors is that the performance of single acalculic subjects may reflect idiosyncratic collections of underlying processes rather than the behavior of a uniform number-processing system. If expertise entails flexibility in problem representation and solution, it may also be the case that the use of subjects with skilled mathematical abilities prior to injury may underestimate the extent of modality-specific processing and of modality-specific acalculias. In sum, the individual difference findings support the encoding-complex view that different people can realize number skills in different ways and advise directly against an approach that tends to ignore this plasticity in favor of an overly simple uniform number-processing system.

Hemisphere specialization and number processing. The research on format effects, acalculia, and individual differences makes brain laterality a central issue in number processing. Several phenomena differentially implicate the left and right hemispheres in number processing from different stimulus formats, such as visual field effects on magnitude and related judgments (e.g., Vaid & Corina, 1989; Klein & McInnes, 1988a, b). The undeniable effects of left-hemisphere damage on number processing clearly implicate the left hemisphere (and presumably verbal processes) in calculation and other aspects of number processing, whereas the research on individual differences implicates imagery and right-hemisphere processes. A right-hemisphere involvement in number processing has also been supported by a variety of other findings relevant to laterality. Annett and Manning (1990), for example, found that arithmetic ability was related strongly to left hand motor skill but not to right hand skill.

Lateralization phenomena have been shown to depend on individual difference factors, again illustrating the complexity and variability of human number processing. As noted earlier, Vaid and Corina (1989) found that lateralization effects depended on familiarity with ASL. Research on soroban users has similarly shown that laterality effects vary with experience. Hatta (1983; Hatta & Ikeda, 1987; cited in Hatta et al., 1989) found that, for soroban experts, finger tapping with the left hand disrupted mental calculation more than tapping with the right hand, whereas the reverse pattern was found for control subjects. This occurred for both auditory and visual problems and was attributed to the greater role of the right hemisphere in mental calculation for soroban experts.

Considerations of brain lateralization effects and number processing demonstrate some of the shortcomings of an excessive dependence on abstract number codes. Advocates of abstract codes must assume that hemisphere-related effects represent noncalculational processes that do not involve the abstract number codes; for example, the effects may be attributed to comprehension processes involved in the translation from specific to abstract number codes. But this account seems implausible for many of the phenomena reported. There is no reason to expect, for example, that comprehension of auditorially presented number problems should vary as a function of abacus expertise, if by comprehension we mean translation from auditory to abstract codes. The findings make more sense if the internal codes used in comprehension and calculation tasks vary as a function of experience. Lateralization issues also raise the important unanswered question of where in the brain the abstract codes are localized. Do they reside in the left or right hemispheres, or in both hemispheres?

Specific Codes in Working Memory and Number Processing

The intimate relation between number processing and working memory also produces evidence relevant to the issue of specific versus abstract number codes. Working memory plays a central role in many aspects of number processing because subjects often must store information while performing other mental operations (e.g., Campbell & Charness, 1990).

Information presented in word problems must be retained temporarily, and even basic numerical calculations (e.g., 4 × 36 = ?) often involve remembering the results of prior operations, what digits must be carried, and other intermediate pieces of information. Recoding and other strategies used to simplify mental calculations (e.g., 4 × 36 = 4 × 3 × 12 = 12 × 12 = 144) also use working memory. There is much evidence to suggest that working memory involves specific verbal or visuospatial codes (Baddeley, 1986; Baddeley & Hitch, 1974), which is easier to reconcile with the specific code view of number processing.

Of particular relevance here, several kinds of evidence converge on the conclusion that working memory number representations are modality- and format-specific. Disruptive effects of concurrent articulation on mental addition (Hitch, 1978) and counting (Logie & Baddeley, 1987; see also Healy & Nairne, 1985) implicate phonological and articulatory working memory codes in number processing. Self-reports and analysis of errors provide evidence for visuospatial working memory codes. For example, abacus experts use a visual representation of the abacus to perform mental arithmetic (Stigler, 1984), and evidence for visual imagery is found in many calculation tasks (Hayes, 1973). The dependence on format-specific number codes (i.e., digits versus words) in working memory is further demonstrated by the finding that shifts from number words to digits (and vice versa) produce as much release from proactive interference as do shifts from one conceptual category to another (Reutener, 1972).

The strong evidence for modality-specific number codes and the need for continuity between calculation and working memory processes present several theoretical challenges for the abstract-modular view. The evidence that modality-specific working memory codes are used in calculation tasks implies that abstract codes cannot be stored briefly and must be translated into a specific code to be retained for even a few seconds. That is, for some reason, "8,043" or "eight thousand and forty-three" can be remembered in working memory as specific codes, whereas the abstract code "{8}10EXP3 {4}10EXP1 {3}10EXP0}" cannot. But it is difficult to conceptualize a physical form for this unusual abstract code that decays instantly when not actively being processed.

Moreover, if calculation uses abstract codes and working memory uses specific codes, then most arithmetic problems would involve the production, calculation, and comprehension systems in repeated translations between specific and abstract codes. During the calculation of 4 × 36, for example, 4 and 6 would first be translated into abstract codes, 24 would be generated and translated back into its working memory equivalent (or separate specific codes for 4 and the to-be-carried 2), 3 would then be encoded abstractly (and perhaps 4 if its abstract code had decayed) and multiplied, and so on. This repeated and arbitrary transformation of codes between specific and abstract forms is unnecessary in the specific-integrated view of number processing, which emphasizes modality-specific codes that are compatible with current models of working memory. That is, the encoding-complex model permits specific visual and verbal codes to serve both as the immediate inputs and outputs of numerical operations and as the transient codes that are temporarily retained between successive operations.

The close relation between calculation and working memory suggests that some acalculic disorders (presumably secondary acalculias, cf. Boller & Grafman, 1983) may arise from disruption of working memory, inasmuch as the phonological store is susceptible to selective disruption with brain injury (Vallar & Baddeley, 1984). A verbal or visual working memory deficit could have strong effects, for example, on multidigit arithmetic problems, such as those used on Jackson and Warrington's (1986) standardized test.

To conclude this examination of the abstractness of number codes, the specific code hypothesis of the encoding-complex theory is supported by various conceptual inadequacies of abstract codes, direct evidence for format-specific phenomena in calculation processes, research on the diversity of acalculias and individual differences in number processing, lateralization issues, and considerations of the intimate relation between working memory and number processing, both of which implicate format-specific number codes. Collectively, the findings raise doubts about the need for abstract codes and challenge the abstract-code account of number dissociations.

INTEGRATED ASSOCIATIVE NETWORKS OR DISTINCT MODULES?

The second major issue that separates the specific-integrated and abstract-modular views concerns the extent to which number processing can be dissected neatly into distinct, functionally independent modules. There are many conceptual and empirical limitations associated with the hypothesis that number comprehension, calculation, and production constitute separate processing modules. Indeed, the various associative, semantic, and linguistic mechanisms involved in number processing are so interconnected and intimately involved in one another's activities that a strong form of modularity is improbable. Also contrary to the general tone of modular theories, number processing appears to involve general cognitive mechanisms rather than special mechanisms idiosyncratic to number-processing modules. The encoding-complex view that number processing involves integrated associative mechanisms is supported by conceptual considerations and by a wide variety of empirical findings.

Conceptual Inadequacies of Modular Views

One conceptual concern about modular views arises from the weaknesses of the abstract-code hypothesis summarized in the preceding section. The modularity hypothesis is intimately related to abstract number representations and indeed may require abstract codes to be viable. For example, McCloskey et al. define comprehension as the activation of underlying abstract number representations and it is only this definition that permits a sharp line to be drawn between comprehension (preaccess to abstract code) and noncomprehension (postaccess to abstract code) processes. Eliminating abstract codes and defining "comprehension" in terms of the activation of a complex of associatively related information, on the other hand, makes it difficult to identify a discrete comprehension module or to draw sharp lines between comprehension, calculation, and production. According to this alternative integrated view, "comprehension" involves activation of varying amounts of the associative complex, with different patterns of activation occurring on different occasions. Similar issues arise in distinguishing between calculation and production modules because production is defined in terms of translation from an abstract to a specific code. Thus, without an abstract code, it is impossible to specify sharp boundaries between comprehension, calculation, and production modules.

A second concern is that modules generally characterize cognitive events at a very high degree of abstraction and may escape specifying the concrete mechanisms needed to carry out the abstract processes. That is, positing black boxes that mysteriously perform some desired operation and calling them modules do not constitute an adequate psychological explanation for number processing, nor do they further our physiological understanding of the underlying mechanisms. Attributing addition, multiplication, and other operations to a "calculation" module, for example, does not explain how arithmetic computations are done (see Seidenberg, 1988, for a similar claim about "box" models of dyslexia). Thus even if such modules as comprehension, calculation, and production did provide a valid taxonomy of number-processing subsystems, they would not constitute an explanation for how numerical tasks are performed.

The limitations of hypothetical modules, in the absence of further detail, can be illustrated for the McCloskey et al. model. The comprehension module, for example, translates from specific number codes (words and digits) into abstract number codes, but we are not told what mechanisms actually perform that operation. Even details that are spelled out, relative to other aspects of the theory, offer only a very abstract sort of "verbal" explanation. McCloskey, Sokol, and Cohen's (1986, pp. 313–314) description of their "Model of Spoken-Verbal-Number Production," for example, describes the first stage of the process in these terms, "the largest power of ten in the number is identified, and on this basis a 'syntactic frame' is generated." But this abstract description fails to address such fundamental questions as: how is the largest power of ten identified, and what is the concrete form of the syntactic frame and how is it generated. Until such information is provided, the explanation is illusory.

Researchers should certainly posit representations and processes that help to explain data, but wherever possible, the hypothetical units should be at least loosely realizable with what is currently known about brain functioning (e.g., associations, excitation, inhibition). Extreme caution should be shown to explanations that are not reducible to known mechanisms in a reasonably straightforward way (e.g., a power of ten operator, a comprehension module, a 'syntactic frame' generator). Indeed, one appealing feature of the classic modularity view was that modules were linked to discrete mechanisms in the brain, at least in principle.

A related concern is that when efforts are made to be more specific about the possible operation of modules, reasonable elaborations raise questions that are somewhat enigmatic from the abstract-modular view. Consider how associative mechanisms might be incorporated to explain the functioning of the comprehension, calculation, and production modules. Comprehension could be explained, for example, by associations between specific and abstract number codes (e.g., 6 → {6}10EXP0, 2 → {2}10EXP0); calculation by associations between abstract codes for number problems and answers (e.g., {6}10EXP0 {X} {2}10EXP0 → {1}10EXP1, {2}10EXP0); and production by associations between abstract and specific codes (e.g., {1}10EXP1, {2}10EXP0 → twelve).

The paradox that arises from such attempts to incorporate associative principles into modular theories concerns the resulting need for artificial restrictions on permissible associations to maintain the strong modularity assumption. To limit calculation to the calculation module, the theory must arbitrarily reject associations between specific number codes for problems and answers. That is, number facts cannot be represented by specific associations (e.g., $6 \times 2 = 12$, six times two equals twelve, 6×2 = twelve). Permitting such associations would have several effects devastating to the abstract-modular model: an abstract calculation system would be superfluous, redundant calculation systems would be possible, and criteria would have to be developed to identify which of the specific or abstract code systems performed the task on any particular occasion. Similarly, to maintain the principle that abstract number codes mediate translation from one specific code to another, the theory must mandate, again arbitrarily, that associations cannot be formed directly between specific codes of different types (i.e., 6 → six rather than 6 → {6}10EXP0 → six). If such codes were permitted, the abstract codes could be bypassed and modular explanations for a variety of tasks would be seriously compromised (see later discussion). There seems to be no principled reason for restricting associations in these ways, other than salvaging the notion of modularity.

Integrated Associations versus Modules

In contrast to the boxes of modular theories, the encoding-complex view explains number processing in terms of concrete associations and retrieval mechanisms that operate on specific visual and verbal codes. These elementary associative codes and processes are the building blocks out of which complex number-processing skills, strategies, and routines are constructed. Numerous findings are consistent with the hypothesis that adult number processing is generally performed by associative networks in which associatively related and recently primed incorrect answers "compete" with the correct answer for selection (Campbell, 1987a, c, 1990c; Campbell & Clark, 1989; Siegler & Shrager, 1984). Evidence for this general class of models comes from patterns of errors and reaction times in both production and verification tasks.

One robust finding is that most arithmetic errors involve associatively related answers. For example, retrieval errors in simple multiplication tend to involve answers from the correct times table and to be close to the correct answer in magnitude (e.g., $6 \times 7 = 48$; Campbell & Graham, 1985). One explanation is that stimuli activate related associations (e.g., "6×7" activates both the correct answer "42" and such associated responses as "$6 \times 8 = 48$"), either by invalid associations between problems and answers or by indirect activation of other problems (e.g., "6×7" primes "6×8," which elicits 48). Consistent with the associative model, priming related incorrect answers disrupts generation of correct answers in production tasks (Campbell, 1987a,b, 1991) and rejection of incorrect answers in verification tasks (Campbell, 1987b; Stazyk, Ashcraft, & Hamann, 1982). These findings support the hypothesis that problems sharing operands are structurally associated in memory and that the multiple internal responses associated with a given number problem mutually interfere with each other's retrieval (e.g., Campbell, 1990c; LeFevre, Bisanz, & Mrkonjic, 1988).

Contextual cues and preceding problems also produce transient changes in the activation level of number representations and associations, as shown by interference effects among related problems. For example, multiplication performance improves when related, interfering problems are removed from the current problem set and deteriorates when related problems are reintroduced (Campbell, 1987a). These findings suggest that associative relatedness and priming by prior problems elevate competing responses, thus increasing errors and reaction times. Campbell and Clark (1989; Campbell, 1991) demonstrated that interference from previous trials exerts an influence over at least 1 min, and possibly much longer. Interference from prior problems also explains why calculation may be easier during the first few minutes of a testing session than later in the session (Chapman, 1915; Phillips, 1916).

Campbell (1990a, b) has recently obtained evidence that interference from preceding problems is modality-specific, suggesting that number facts are represented by associations that connect modality-specific representations, rather than by a single calculation module. Mutual interference was examined among alternating digit and number-word problems. For both addition and multiplication, errors were most likely to derive from earlier problems presented in the same modality as the problem eliciting the error. That is, errors on digit problems matched correct answers to previous digit problems more often than they matched correct answers to previous word problems. Errors on word-format problems, on the other hand, matched answers to previous word problems more often than they matched answers to recently presented digit problems. Selective interference as a function of number format suggests that number–fact retrieval via digits and words is mediated by format-specific associations, rather than by a shared calculation module involving common abstract number codes.

The encoding-complex hypothesis is that various modality-specific codes relevant to diverse facets of number processing are connected in an integrated associative network. Such a framework accounts for the interference effects observed in number processing and, moreover, suggests that inhibitory mechanisms might play a central role in controlling the noisy and error-prone activation that could result from such networks. It has long been widely accepted in neuropsychological circles that inhibitory mechanisms play a central role in fine-tuning the activity of the human brain, and indeed the classic role of inhibitory processes in perception and other psychological domains is to enhance contrast and suppress noise. We believe that this same function, and others, is performed by inhibitory processes in human cognition (cf. Clark & Campbell, 1990) and that arithmetic tasks may be particularly sensitive to inhibition and related interference processes because people acquire many interrelated associations among a relatively small set of modality-specific number representations (Campbell, 1990b). Modular theories with well-defined boundaries between different phases of processing would presumably require less emphasis on inhibitory mechanisms.

Initial evidence for retrieval inhibition in mental arithmetic was obtained by Campbell and Clark (1989; see also Campbell, 1990c). Using simple multiplication problems, Campbell and Clark showed that whereas answers to previous problems overall appeared as error responses to later problems more frequently than expected by chance (the *positive error priming* described previously), answers to immediately preceding problems appeared as errors less frequently than expected (*negative error priming*). Negative error priming suggests that the answer from the preceding trial was inhibited, and indeed the results were fit well by excitatory and inhibitory exponential curves with different decay rates. Further supporting the inhibition interpretation, Campbell (1991) showed that correct reaction times were faster when a potentially interfering (i.e., related) product was produced and presumably inhibited on the preceding trial than when an unrelated problem occurred on the preceding trial. Inhibition is probably involved in more than controlling interference effects between closely spaced retrievals, and it may even prove difficult to build concrete associative networks for some arithmetic facts without using inhibitory connections. According to this view, interference effects represent failures to inhibit competing responses, which has several implications for interpretation of number dysfunctions.

Associative Mechanisms and Acalculia

The encoding-complex hypothesis that excitatory and inhibitory associative processes contribute collectively to the performance of number tasks implies a somewhat different view of acalculia than that presented by the modular theory (see next section). Rather than assuming that brain damage would selectively impair comprehension, calculation, or production, the integrated associative model suggests that partial dysfunctions might be associated with selective damage to inhibitory processes. This conclusion follows from the model and from evidence that inhibitory processes may be particularly susceptible to disruption. Inhibitory neurotransmitters, for example, are more vulnerable than excitatory neurotransmitters to hypoxia and other brain insults (e.g., Krnjevic, 1983; Roberts, 1987). Consequently, inhibitory processes are more likely to be damaged than excitatory processes, which could remain relatively intact.

Given the hypothesized role of inhibition to reduce noise and suppress competing responses (e.g., Campbell, 1990c, 1991; Clark & Campbell, 1990; MacKay, 1987), enhanced interference effects should be a common symptom of brain injury in numerical tasks. Consistent with this view, Campbell and Clark (1988) demonstrated that the digit-reading errors of McCloskey et al.'s patient, HY, involved intrusions of numerically and visually similar responses. The hypothesis is that convergent perceptual and calculational associations activate competing responses, just as in normals (e.g., den Heyer & Briand, 1986), but that damaged inhibitory mechanisms generate inadequate suppression. Similarly, the errors produced to simple multiplication problems by brain-damaged patients (e.g., McCloskey, Sokol, Cohen, & Ijiri, 1986; Sokol, McCloskey, Cohen, & Aliminosa, 1991) are exaggerations of the patterns produced by competent adult subjects tested under speed pressure (e.g., errors are more common for larger number problems and tend to be associatively related to the problem). A straightforward explanation for these phenomena follows from the integrated associative view that we have proposed; namely, activation spreads to associated responses much the same as in normal subjects, but acalculics experience an impaired capacity to inhibit or control the competing responses.

Reevaluation of Modular Architectures and Dissociations

One benefit of the modular view, if it were correct, would be that cognitive dysfunctions in number processing could be explained by selective damage to specific modules. A basic requirement for such use of the abstract-modular theory and for its evaluation is the identification of which modules are relevant to various number-processing tasks. However, conceptual and empirical considerations show the difficulty of identifying in any principled way the distinct theoretical modules that are involved in different numerical tasks. Here we illustrate the problems for several tasks that have been used by McCloskey et al. and others to make inferences about dysfunctional modules.

A consideration of basic counting skills and their relation to calculation and number-production skills demonstrates that the boundaries between number comprehension, calculation, and production are fuzzier than suggested by proponents of the abstract-modular view (e.g., McCloskey et al., 1985; Sokol et al., 1989). Although nothing specific has been claimed about which module performs counting, Sokol et al. (1989, p. 108) stated that counting is not a normal function of the calculation system. Putting counting processes "outside" the calculation system creates problems for modular theories, however, because children and sometimes even adults (Svenson, 1985) use counting strategies to produce and verify number facts and to perform other tasks explicitly assumed by McCloskey et al. (1985, 1986) to involve the calculation system. Thus, "calculation" could be performed by strategies that involve comprehension or production systems and presumably could be disrupted by damage to those "non-calculational" systems. Neither can counting be placed just in the calculation module, however, since research also indicates that counting relies on phonological articulatory codes (e.g., Seron & Deloche, 1987; Healy & Nairne, 1985; Logie & Baddeley, 1987), which localizes some aspects of counting in the verbal comprehension or production modules.

Magnitude judgment tasks are equally difficult to dissect into neat components. McCloskey et al. (1986) use magnitude judgments as fundamental tests of "comprehension," and Sokol et al. (1989) indicated that the abstract quantity codes generated by the putative comprehension system "reflect numerical nearness relationships" (p. 108). Despite these hints about the modules involved in magnitude comparison, it is difficult to specify clearly what modules do contribute to magnitude comparison. Instead there seem to be myriad ways in which magnitude judgments could be performed and inadequate evidence at present to choose among these alternative accounts or to reject the hypothesis that the task is performed in different ways by different people or in different contexts.

Magnitude judgments could be based on counting string associations (six occurs later than five in counting), which implicates verbal production processes, or on visual representations of number lines (six is to the right of five), or on calculation relationships (e.g., five *plus* one is six). It seems somewhat tenuous, therefore, to use success and failure at magnitude judgment to assess the integrity or dysfunctioning of a single comprehension mechanism.

The difficulties associated with identifying the contribution of comprehension, calculation, and production processes to number-processing tasks are also demonstrated by a careful analysis of the arithmetic-verification task (e.g., 4 + 8 = 15, true or false?). McCloskey et al. (1985, 1986) seem to assume that this task assesses the integrity of the calculation system without involving the production system. One theory of verification, however, proposes that subjects generate an answer for the problem and check it against the presented answer (cf. Campbell, 1987b; Stazyk et al., 1982; but see Zbrodoff & Logan, 1990). If this generate–compare process involved production of verbal or visual representations for the answer, then errors in the production module could lead to incorrect verification responses. There is also evidence that magnitude (Stazyk et al., 1982) and odd–even status of the presented answer (e.g., Krueger, 1986; Krueger & Hallford, 1984) contribute to specific verification strategies. We have already shown, however, that magnitude judgments need not involve the putative calculation system. Thus, correct or incorrect verification performance may reflect processes external to the nominal calculation system.

Similar considerations compromise interpretation of other possible performance dissociations in acalculia. For example, better performance on sum verification relative to sum production might be viewed as evidence of a deficit in production processes. This conclusion only follows, however, if the identical calculation process is involved in the verification and production tasks. But as just noted, subjects can exploit odd–even and other numerical relationships to reject false answers in the verification task, whereas these strategies are not possible when the task is to generate an answer. Furthermore, brief preexposure (200–300 msec) of the correct answer sharply reduces retrieval errors on simple arithmetic problems (Campbell, 1987b, 1991), and this priming effect may well operate when the correct answer is presented in a verification task. Thus, differences between verification and generation could reflect a failure to retrieve the correct answer (a "calculation" failure in modular theories) rather than a failure to translate the correct answer into its verbal form (a "production" failure). Successful verification therefore provides imperfect information about the integrity of calculation processes involved in generation tasks.

Readers may perceive some inconsistency in our preceding discussion of the verification task. We first claimed that production processes might contribute to performance on verification tasks and then argued that priming might reduce the contribution of production processes to verification. It is important to note that our aim here is to state not how verification is done, but rather how it might be done. The tendency of modularity theorists is to adopt a conceptualization of number-processing tasks that fits their modular scheme. But alternative conceptualizations are possible, and even apparently inconsistent positions (e.g., verification involving production or nonproduction) might all be true for different people or in particular contexts. More generally, the fact that tasks can be performed in diverse ways using a variety of representational media and processes means that a simple interpretation of task-specific dissociations will often be impossible.

Even a deceptively simple number-naming task demonstrates several difficulties associated with inferring cognitive dissociations. McCloskey et al. (1986) localized the number-naming deficit of patient HY in verbal number-production processes because of adequate performance on a variety of "comprehension" and "calculation" tasks. Campbell and Clark (1988), however, demonstrated that Hy's number-word substitution errors (e.g., stating "six" for "2") were predicted by visual and arithmetic similarity (numerical nearness and odd–even agreement). Although such results are possible given a pure production deficit (see next paragraph), they are also consistent with the competing hypothesis that failure to inhibit visually similar codes (a "comprehension" process?) and semantically related responses (a "calculation" process?) contributed to the naming deficit. These early processing deficits may not be revealed by tasks that prime correct answers (e.g., matching tasks) and that therefore place less demands than naming on retrieval processes.

Not only is it difficult to localize in a principled way the overall dysfunctions demonstrated by acalculic subjects, but also patterns of results do not support specific inferences about localized processes drawn by modular theorists. For example, McCloskey et al. (1986) concluded that the pattern of number-naming errors of HY permitted inferences about the organization of responses within the dysfunctional number-word production system. Specifically, they concluded that because HY tended to produce naming errors within the same number class (ones, teens, tens) as the correct response, the lexicon of the production module was organized by tens. But nothing prevents this pattern of errors from arising outside of the production system, even if the deficit were specific to "production" (e.g., a failure to suppress interfering responses). For example, the calculation module may activate related values within the same number class as the correct response, resulting in subsequent activation of these competing responses in the production system. The evidence that visual and numerical factors influenced HY's number naming (Campbell

& Clark, 1988) is consistent with this alternative interpretation. Thus, inferences about specific modules based on patterns of errors and nonerrors must be made cautiously.

Finally, dissociations based on format effects on number processing are particularly relevant in the present context, because proponents of the abstract-modular theory use format-related dissociations in brain-damaged patients to infer the functional locus of a patient's deficit and to deduce the architecture of number processing. The McCloskey et al. (1985) theory, for example, makes a strong distinction between comprehension deficits (i.e., failure to access the correct semantic-quantity code) and calculation deficits (i.e., failure to perform the correct computation given appropriate abstract codes). The logic is as follows: If a calculation can be performed successfully on one type of stimulus (e.g., digits), but not on another (e.g., number words), then the comprehension subsystem associated with the latter must be impaired. This conclusion follows because unimpaired calculation for a single format confirms that the abstract calculation process is intact (cf. McCloskey et al., 1985, p. 178).

The reasoning is not valid, however, when the "calculation" processes that follow stimulus encoding vary with format. If different number formats selectively activate distinct number representations and associations rather than a single abstract code, as suggested by evidence reviewed earlier, then successful judgment, calculation, or production for one stimulus modality only validates processes specific to that format. For example, the ability to use visual codes to make magnitude judgments for digit stimuli would not confirm that modality-specific comparison processes are intact for phonological codes, which may be more salient for number words. Thus, dysfunctional comparisons for words could be due to either a comprehension or a comparison deficit. Thus, the logic for interpreting performance dissociations is put in doubt by comparison or calculation processes that vary with format. At the very least, such considerations challenge the uncritical use of format-related dissociations to localize deficits in discrete number-processing modules.

Semantic Structure of Verbal Number Systems

An additional weakness of the modularity assumption is that calculation processes (e.g., knowledge of simple arithmetic operations and facts) are functionally separated from language processes that require calculational knowledge to operate (i.e., syntactic and lexical mechanisms involved in the comprehension and production modules). That is, the separation of calculation from number meaning and linguistic processes ignores the close relationship between arithmetic operations and number-naming processes (Menninger, 1958/1977). The verbal-number production system proposed by McCloskey et al. (1986) does incorporate semantic roles defined by arithmetic (e.g., the abstract code generated for the ten's position in "40," {4}10EXP1, implies the multiplicative function 4 × 10), but assumes that this knowledge is separate from the corresponding procedural and factual knowledge of arithmetic used to perform calculation and other numerical tasks. There is evidence, however, that number-processing strategies and calculation skills are at least partially determined by the semantic structure of verbal-number systems. Although the names for "primitive numbers" (i.e., numbers which are directly named in a given language) are often arbitrary, for example, some languages contain basic number names that imply the corresponding quantity (e.g., the word for "five" may be based on the word for "hand"; Fuson & Kwon, 1990). Furthermore, in many languages, numerals (e.g., 1026) are converted to number-word sequences (e.g., one-thousand and twenty-six) by production rules that correspond to arithmetic relationships (i.e., 1 × 1000 + 20 + 6) (Boden, 1988).

Computational analyses of number production (Power & Longuet-Higgins, 1987) provide further evidence for the claim that knowledge of calculation relationships is intertwined with syntatic rules for verbal-number comprehension and production. A program developed by Power and Longuet-Higgins "learns" to convert numbers presented in digit form to verbal-number names in several languages. It memorizes names for primitive numbers and infers grammatical rules for generating names for non-primitive numbers based on a set of semantic formulas representing arithmetic relationships (i.e., addition, subtraction, and multiplication). For example, 94 is named "ninety four" in English (i.e., 90 + 4) and "quatre vingt quatorze" in French (i.e., 4 × 20 + 14). Power and Longuet-Higgins propose that the base grammar for number naming and counting in all languages is based on semantic roles provided by simple arithmetic. The various ways in which the same number may be named also demonstrates that number-naming systems involve intrinsic knowledge of arithmetic relationships. For example, the number 2200 may be stated as "two thousand two hundred" or "twenty-two hundred," depending on which arithmetic relationships are used to parse and interpret the string of digits. As Boden (1988, p. 198) concludes, "the 'syntactic' rules for forming numerals . . . cannot be stated without referring to *semantic* content: the arithmetical notions of sum, product, difference, and primitive number" (author's italics).

Beyond this evidence that number-naming systems use knowledge of arithmetic relationships, number representation and calculation skills also appear to be at least partially determined by linguistic structure. Fuson and Kwon (1990) reviewed studies showing that linguistic structure determines how children use tokens to represent quantities. For example, the tens position is usually explicit in Asian number-naming systems (e.g., 34 is named "three ten four" in Chinese), but implicit in European languages. Children who learn to name the tens are more likely to represent numbers with tens and unit tokens, whereas age-matched children who learn a system that does not name the tens are more likely to count out units. Different number-naming systems also influence how children learn to perform single-digit and multi-digit addition and subtraction (Fuson & Kwon, 1990). These observations challenge a model which sharply separates calculation from the grammatical processes of verbal language. For example, it is not clear why European and Asian children would differentially recognize the role of tens in 34, if both represent it as {3}10EXP1, {4}10EXP0.

Continuity of Numerical and Nonnumerical Processes

The issue of modularity is also relevant to the general question of the continuity between number-processing mechanisms and nonnumber mechanisms. Although McCloskey et al. drew a general parallel between their syntactic model for number production and models of grammar, their model depends heavily on structures and processes that are specialized for number processing. The 10EXP*n* operator that underlies the abstract number codes, for example, appears to have limited generality. Similarly the calculation module and the specific processes involved in translating abstract quantities into number words (e.g., generation of numerical syntactic frames) are specific to number processing.

The encoding-complex view, on the other hand, suggests that normal and dysfunctional number processing often involve basic cognitive mechanisms that are generic in nature rather than unique, number-specific mechanisms. That is, the encoding-complex theory explains number processing in terms of the same modality-specific representations and associative mechanisms used to explain nonnumerical aspects of cognition, whereas the abstract-modular view hypothesizes an architecture that implies relative independence from other cognitive domains: abstract number codes, a specific calculation module, and a specialized number-processing system.

Consistent with our connected view, the same issues about abstractness and modularity surface in numerical and nonnumerical domains. The question of abstract versus format-specific semantic codes, for example, is important in research on bilingualism and on picture and word processing (e.g., Clark, 1987; Glucksberg, 1984; Vaid, 1988). We have also seen that number processing involves associative mechanisms similar to those found in traditional semantic memory research with verbal and nonverbal materials (e.g., Anderson, 1983; Gillund & Shiffrin, 1984; Paivio, 1986). Moreover, common mechanisms of associative interference appear to mediate memorization and retrieval of numerical and nonnumerical information (Graham & Campbell, in press), and general problem-solving strategies and pitfalls (e.g., effects of set; Greeno, 1978) are similar for numerical and nonnumerical problems.

The inhibitory components of our integrated model are also general mechanisms that have been proposed in several models of associative memory (e.g., Anderson, 1983; Gillund & Shiffrin, 1984) and cognitive skill (MacKay, 1987). Active suppression of alternative responses contributes to selection of targets in selective attention (e.g., Tipper, 1985) and many other tasks. Thus, inhibition could be centrally involved in the "selection" of a single response or action from among multiple possible responses, with the dominant response that eventually emerges doing so through inhibition of competing responses. This generic model would apply to both numerical and nonnumerical tasks.

Emphasizing continuities between number and nonnumber processing raises questions about acalculics's nonnumerical performance, which can provide insights into their numerical deficits. The integrated view that number skills do not comprise a separate cognitive "system" suggests that acalculia can reflect dysfunctions in general mechanisms that should affect other cognitive tasks. Perhaps reflecting a belief that number processes are unique, acalculia researchers sometimes include minimal information about performance of their patients on nonarithmetic tasks or relegate such information to an appendix with little or no discussion. Although McCloskey et al. (1986) concluded that HY had a deficit in retrieving lexical items within the number-word production system, for example, data presented in an appendix suggested that HY was even more impaired at picture naming.

The case of digit and picture naming illustrates that consideration of parallels between numerical and nonnumerical processing can shed light on both domains. We hypothesized that HY's impaired digit naming reflected failure to inhibit visually and numerically similar responses. Inhibition among competing responses has also been used to explain interference effects in picture naming, such as increases in reaction time with number of different names (i.e., response uncertainty) (e.g., Clark & Johnson, 1990; Johnson & Clark, 1988; Paivio, Clark, Digdon, & Bons, 1989). Failure to suppress competing responses therefore explains both HY's digit naming and picture naming dysfunctions in terms of common underlying processes. The additional fact that Hy may have been more disturbed at picture than digit naming can also be explained by this interference mechanism, because pictures have higher uncertainty than do digits. Mills, Knox, Juola, and Salmon (1979) have demonstrated that aphasics show stronger naming deficits for high uncertainty pictures than for low uncertainty pictures analogous to digits.

To summarize our discussion of modularity, there are many conceptual and empirical grounds for questioning the assumption of well-defined number-processing modules and for accepting the alternative view that number processing is carried out by integrated associative networks acted on by generic excitatory and inhibitory mechanisms. Support for the integrated associative model includes: dependence on abstract codes and other conceptual inadequacies of modular views, much evidence for the role of integrated associative networks in number processing, the mechanistic explanations for acalculia provided by associative processes, empirical and in-principle difficulties associated with relating modular cognitive architectures to number-processing dissociations, evidence that the semantic and verbal structures of numbers (i.e., aspects of calculation and production modules) are intimately related, and continuities between numerical and nonnumerical processing.

GENERAL DISCUSSION AND CONCLUSIONS

We have examined in some detail two central and related questions about number processing. Are numbers represented and processed primarily as a single, abstract quantity code or as multiple, format-specific codes? Is number processing usefully characterized by a relatively small set of sharply distinct modules each designed for some specialized purpose or by an integrated collection of shared representations and basic associative mechanisms that participate in many different tasks? Our conclusion is that the assumption of independent number-processing modules and a special calculation system that acts solely on abstract number representations, as proposed by McCloskey et al. (1985, 1986; Sokol et al., 1989), greatly oversimplifies number processing. The encoding-complex alternative, which emphasizes integrated mechanisms operating on specific number codes, seems better suited to the complexity and richness of number processing and has been found preferable on a variety of conceptual and empirical grounds.

We have argued that the two central assumptions of the abstract-modular theory—that number knowledge is based on a single type of abstract code and that number processing can be divided neatly into a small number of separate modules—are not well motivated and are put in question by empirical findings in normal and dysfunctional number processing. Moreover, the box model approach to explanation provides limited insights about the mechanisms that underlie numerical skills and the subtle ways in which skills may break down with brain insult.

Although we have taken a strong stand in this paper on the contrasting perspectives of the specific-integrated and abstract-modular views, there is room for compromise between the two positions. Although an extreme form of modularity is rejected, the encoding-complex view does allow for aspects of number processing to become integrated as functional units. For example, the semantic relations involved in converting a string of digits to the appropriate number name may become *proceduralized* with practice (Anderson, 1982), producing a functional unit analogous to the "syntactic frame" model proposed by McCloskey et al. (1986; Sokol & McCloskey, 1988). That is, the underlying semantic structures may no longer be processed explicitly in simple naming tasks. In a restricted sense, then, this specific skill could involve a relatively autonomous cognitive routine, perhaps especially if inhibitory processes became so ingrained as to produce a type of functional independence akin to that envisioned by modular theorists.

We also recognize that the encoding-complex theory is currently incomplete, as is the abstract-modular view, and that this incompleteness makes a definitive choice between the two approaches impossible. Nonetheless, we have shown that the encoding-complex view presents a clear alternative to the assumptions of abstract codes and modular number functions and makes nontrivial predictions about the nature of number processing. The theory also provides clear directions as to how detailed, mechanistic, and empirically well-founded models of specific number-processing tasks can be developed and is generally consistent with much cognitive and neuropsychological research on both number and nonnumber processing. Modality-specific number processes, for example, are compatible with neuropsychological knowledge about the varied brain insults and behavioral consequences that are labeled acalculia and with current knowledge about individual differences and lateralization effects in number processing.

In conclusion, the construction and depiction of theories would certainly be easier for cognitive psychologists and neuropsychologists studying number skills if both the abstract-code hypothesis and the assumption of modularity of number processing were substantially correct. Certainly, integrated networks of specific codes cannot be displayed as neatly as modular theories of number processing, which lend themselves so well to depiction by simple box models. But we see no reason to presuppose that the brain is functionally organized in a neat manner that facilitates description of theories, nor even their creation. This caveat seems especially germane in the context of number skills and acalculia given the diversity of information-processing strategies that can be brought to bear in numerical problem solving. We have outlined an approach to number processing that emphasizes modality-specific representations, integrated networks, and excitatory and inhibitory associative mechanisms. This theoretical approach accommodates diverse phenomena in research on normal and dysfunctional number processing, is consistent with theories of nonnumerical phenomena, promotes theoretical and empirical collaboration between neuropsychologists and cognitive psychologists, and identifies numerous directions for future research into the mechanisms that underlie normal number processing and acalculia.

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