Inhibitory Mechanisms in Normal

and Dysfunctional Number Processing

James M. Clark

University of Winnipeg

Running Head: Inhibition and Number Processing

Clark, J. M. (1992). Inhibitory mechanisms in normal and dysfunctional number processing. In J. I. D. Campbell (Ed.), *The nature and origin of mathematical skills* (pp. 411-456). New York: Elsevier.

Summary

Inhibitory mechanisms can play central roles in associative theories of number processing, including context-sensitive activation of overlapping associative networks and exaggeration of differences between competing nodes. I review conceptual and empirical considerations that implicate inhibitory mechanisms in these important functions, not only in number processing but also in other cognitive domains. Many interference and related phenomena are shown to be consistent with the proposed roles for inhibition. Central roles for inhibition have many implications for such correlates of normal and dysfunctional number processing as physiological and psychological indicators of inhibitory functioning, brain damage, childhood development, and aging. More generally, associative theories based on inhibition and excitation encourage the development of unified mechanistic explanations soundly rooted in diverse empirical phenomena.

Inhibitory Mechanisms in Normal and Dysfunctional Number Processing

James M. Clark

University of Winnipeg

This paper examines empirical and rational support for the hypothesis that inhibition serves major roles in the associative mechanisms that underlie number processing, and proposes some novel implications of that hypothesis for correlates of normal and dysfunctional performance on numerical tasks. The major roles discussed are context-sensitive activation of problem answers and, more briefly, enhancing differences in activation between competing answers. The implications have to do with such inhibition-related correlates of number processing as age and brain damage.

In the main part of the paper, I show that inhibitory associative structures based purely on operand-driven activation can selectively activate correct answers, even for ambiguous problems whose operands are both connected to more than one answer (e.g., 4 and 6 are connected to 12, 24, and 36). Operand-driven selective activation has important implications for number processing phenomena and for the assumption that entire number problems are represented by distinct configural mental codes (e.g., Campbell & Graham, 1985). These configural representations, also called gestalt or holistic codes, are presumed by their proponents to exist in addition to separate representations for the components of the problem (i.e., operands, operators, and answers).

A second section examines a related role for inhibition, to enhance existing differences in levels of activation. Inhibitory mechanisms can exaggerate even small differences in activation levels, often leading to the complete suppression of what would otherwise be strongly competing responses. I then review what the hypothesized prominence of inhibitory mechanisms implies about individual differences and other correlates of normal and dysfunctional number processing. Variables from quite diverse areas of psychology are shown to implicate inhibitory mechanisms, and hence to have potential relevance to arithmetic and other numerical tasks. Before focusing on these inhibitory processes, I briefly describe the basic characteristics of associative models, especially as they relate to number processing.

Associative Models of Number Processing

In essence, associative models of number processing posit mental representations for digits, number-words, operators (e.g, x, +), and other number-related entities. These representations are interconnected in a complex network that permits codes to collectively influence each other's levels of activation. Excitation and inhibition spread in parallel among the individual units to produce the successive patterns of activation that underlie number-related experiences and behavior.

To illustrate, stimulation of the codes for 9, x, and 3 would activate other relevant codes via excitatory connections, including verbal codes for "nine," "three," and "twenty-seven." In addition to these excitatory connections, some associative models include inhibitory associations that can reduce activation in connected codes. For example, the codes for such competing verbal responses as "eight," "twelve," and "eighteen" might be suppressed when the codes for 9, x, 3, and their positively related codes are activated.

The basic assumption of associative models is that spreading excitation and inhibition among a suitably conceptualized network of representations will reproduce relevant findings in human behavior and experience, for example, production of the correct answer on most trials, various types of errors, and latency data. Strong versions strive to explain number-processing purely in terms of a limited set of mental representations, associative connections, and spreading excitation or inhibition.

Associative models can be contrasted with models cast in terms of such high level symbolic processes as strategies and rules (e.g., IF 3 and 7 THEN 21). Advocates of associative models recognize that much human behavior can be described at these higher levels of abstraction, but constrain their psychological explanations to purely mechanistic processes and try to avoid propositions and assumptions that cannot, at present anyway, be characterized in terms of identifiable concrete mechanisms. This emphasis on basic mechanisms also means that associative models involve domain-independent processes, as opposed to rules or procedures designed for specific domains, such as number processing. Finally, the associative perspective presented here emphasizes explicit connections among coherent representations for meaningful objects and events (e.g., number words, images of digits), as opposed to the distributed representations of some neural net approaches.

This associative framework is the foundation for the specific functions and implications of inhibitory processes examined in this paper. In general, I argue on both rational and empirical grounds that inhibitory associative mechanisms are necessary to explain certain phenomena in number processing or, at the very least, explain the phenomena more parsimoniously and elegantly than models that ignore inhibition. Important roles for inhibition in theories of number processing are expected, of course, given the ubiquitous presence in the brain of such inhibitory neurotransmitters as GABA (Roberts, 1987), and the increasing use of inhibitory theoretical mechanisms to explain phenomena in selective attention (Tipper, 1985; Walley & Weiden, 1973), animal learning (Williams & Clark, 1992), aging (Hasher & Zacks, 1988), childhood development (Bjorklund & Harnishfeger, 1990), and diverse other areas of psychology including number processing (Campbell & Clark, 1989).

The specific functions and implications of inhibitory mechanisms are considered here detached from other essential and often unresolved features of associative models; hence, the present paper does not provide a complete associative model for number processing, and is sometimes quite speculative even about the isolated mechanisms that are discussed. More fully developed associative models are available for such numerical tasks as multiplication (e.g., Campbell & Oliphant, this volume), and some of the general issues related to this and contrasting classes of theory have been examined elsewhere (Campbell & Clark, 1989; Clark & Campbell, 1991).

The primary role of inhibition emphasized here is providing context-sensitive or selective activation of responses, a demanding task in the highly overlapping associative networks involved in number processing.

Inhibition and Context-Sensitive Activation

A major challenge for associative models of number processing is to construct associative networks such that correct answers are activated primarily when the appropriate operands occur together. To achieve this specificity of activation, the associative networks that underlie number processing must be highly context sensitive or selective.

The requirement that spreading activation be context sensitive is demonstrated most clearly in the case of ambiguous problems whose operands are both connected to more than one shared answer. Because 4 and 8 are each connected to 24 as well as to 32, for example, simple excitation from 4 and 8 would activate both answers equally. Operand-based activation must somehow be context sensitive in order to produce the correct answer; that is, 4 should activate 24 when 6 is the co-operand and 32 when 8 is the co-operand.

Although ambiguous problems provide the starkest examples of context-sensitivity, the fact that all numerical operations involve associations among a small set of operands means that selective activation is important even for problems that are not completely ambiguous. The multiplication facts 3x5=15 and 3x6=18, for example, would be stored well by a network in which activation from 3 spreads primarily to 15 when 5 is the co-operand of 3 and to 18 when 6 is the co-operand of 3. Similar conditional activation would be required for the rest of the 3 times-table and for the times-tables for other multiplication facts.

A third example of the need for context-sensitivity arises from different arithmetic operations that contain the same operands, and even the same operand combinations. To illustrate, 3x5 should activate 15, whereas 3+5 should activate 8. Thus the destination (i.e., answer) to which activation spreads from the operands and their combination depends in large part upon the presence of a specific arithmetic operator, which presumably acts as a contextual or modulating stimulus that steers activation from the operands to the appropriate response.

These three examples demonstrate the necessity of context-sensitive activation in associative models for arithmetic facts. Alternative methods to provide such contingent activation are discussed later (e.g., unique composite representations for problems, as in Campbell & Graham, 1985, or of problems and answers, as in Campbell & Oliphant, this volume), but I first want to examine whether and how pure associative networks that posit only operand and answer representations could achieve the requisite selective activation. Inhibitory mechanisms in fact provide a powerful tool for generating just the sort of context-sensitive activation required to model the preceding behaviors and other examples of conditional responding.

A Disinhibition Model for Context-Sensitive Activation

Context sensitivity is a generic problem in many areas of psychology. One area in which the basic associative mechanisms underlying selective activation have been investigated is Pavlovian conditioning. There are a variety of conditioning procedures in which training and learning are highly dependent on context, and such tasks show surprising relevance to the associative processes of interest here. I describe briefly two types of conditional learning, occasion setting and patterning, and an inhibition-based mechanism to explain such learning (for further discussion, see Williams & Clark, 1992).

In occasion setting, animals are exposed to a conditioned stimulus (CS) that is only followed by an unconditioned stimulus (US) when the CS itself is preceded by a third stimulus, called a feature (F). The US does not follow a CS that occurs without F. This conditional learning can be described notationally as: F->CS+ versus CS-, where -> indicates the temporal relation between F and the CS, and + and - indicate the presence and absence of the US. After training, animals emit the conditioned response in the presence of F->CS and do not emit the response in the presence of CS alone. A variety of phenomena have demonstrated that F is qualifying the stimulus properties of the CS, and not simply controlling the response directly;

for example, F can be a positive occasion setter for one CS and a negative occasion setter whose absence signals the US for a second CS. Thinking of the CS as one operand (e.g., 8), F as a second operand (e.g., 4), and the response as the correct answer (32), demonstrates that occasion setting involves exactly the kind of context-sensitive activation required to learn arithmetic facts.

Patterning (or configural learning) is a second type of conditioning that also shows striking relevance to the associative structures that underlie individual number facts. In positive patterning, animals learn to respond when both elements in a compound stimulus occur and to suppress responding when individual elements in the compound occur alone. This form of learning is symbolized as AB+, A-, and B-. Animals readily learn such conditional responding. If we think of A and B as operands and the answer as the correct response, patterning again shows just the kind of context-sensitive activation involved in ambiguous or other arithmetic problems.

Although a variety of models have been proposed to explain feature setting and patterning (several relevant to analogous models for number processing are mentioned later), I focus here on an inhibition approach suggested recently by Williams and Clark (1992). Simple associative networks to correctly perform occasion setting and patterning are presented in Figures 1A and 1B. Two components of the highly similar models are critical for producing selective activation. First, the models assume that CSs acquire both excitatory and inhibitory connections to mental representations of the US (the mental code for the US is hypothesized to elicit the response). Theoretically, inhibitory connections develop because there are identifiable occasions when CSs are not followed by the US. The second component of the model is a modulating stimulus (F in occasion setting or the other stimulus in patterning) that inhibits the CS->US inhibitory pathway. This disinhibition reveals the excitatory connection that was previously masked by the now-suppressed inhibitory connection. Williams and Clark (1992) suggested that disinhibition models could explain various context effects in animal learning and reviewed some findings consistent with the proposed models.

Insert Figure 1 about here

Disinhibition is an excellent way to conceptualize and model forms of context-sensitive activation in addition to those that occur in Pavlovian conditioning. To illustrate with an everyday example, people who regularly drive different cars must produce vehicle-specific acts to control the lights, the wipers, and other functions. Perfect responding would require highly selective associative mechanisms, and even minimal performance would involve considerable contextual control over responding. The ability to switch between appropriate sets of responses, including occasional errors, is modelled elegantly by the disinhibition hypothesis, which maintains for this example that vehicle-specific responses are inhibited unless released by cues that one is in the appropriate car.

Clearly any inferred relation between human number processing and such basic behaviors as Pavlovian conditioning or even simple motor responses must be drawn carefully. Nonetheless, the conditional learning phenomena described here are isomorphic to context-sensitive effects in multiplication and other number facts, suggesting that disinhibition might help to explain selective activation in arithmetic.

<u>Inhibition and Disinhibition of Arithmetic Answers</u>

Thinking about how calculation facts are learned suggests that inhibitory mechanisms may develop in precisely the ways just described to explain context effects in Pavlovian conditioning. Multiplication and other arithmetic operations are learned initially by exposure to pairs of operands and their answers, sometimes with prior production of the answer. That is, people study such problems as 3x5=15 or 3x5=?, and subsequently receive feedback on the correctness of their answers. It seems evident that one effect of repeated trials will be to strengthen excitatory associations between the individual operands and the correct answer.

Presentation of the problem 3x5=15, for example, will strengthen the connections between 15 and the operands 3 and 5. Later presentations of 3 and 5 thereafter result in spreading excitation from the presented operands to the answers with which the operands have been paired.

Inhibition of incorrect answers. What may be less evident than these positive connections is that such experiences also support learning something about the relation between each operand and the answers for multiplication problems that are not correct on the current trial. The problem 3x5=15, for example, informs the system not only that 3 should activate 15, but also that 3 should not activate other table-related answers (i.e., 6, 12, 18,...), at least not when 5 is also present. Ignoring for the moment the conditional nature of the preceding statement, this analysis suggests that the mental network will acquire or strengthen negative connections between each operand and answers that are incorrect on the current trial. This suppression is especially important for answers to table-related problems involving each of the operands (e.g., 3x6=18, 6x5=30), because these related answers receive any unsuppressed excitation emitted by the individual operands. That is, it would be particularly useful if this irrelevant excitation were suppressed when the appropriate co-operands were not present.

Negative associations between operands and incorrect responses directly implicate inhibitory processes. Panel A in Figure 2 presents an idealized representation for part of a network in which each operand acquires both excitatory and inhibitory connections to table-related answers. The inhibitory connection depicted in Figure 2A seems intuitively reasonable. There are innumerable occasions on which 3 occurs in the absence of 15 and when production of 15 would be incorrect. Indeed the vast majority of times that individual operands such as 3 occur, they involve answers other than the correct answer for one particular problem (15 in the present example). These numerous "negative" trials provide many opportunities for inhibitory learning between operands and answers, and it would perhaps be surprising if such inhibition did not form, especially for incorrect answers from the same times-table.

Insert Figure 2 about here

Although Figure 2A and most of the remaining discussion characterize inhibition in terms of a single connection from the operand to the answer, the detailed underlying mechanism is almost certainly more complex. In particular, there are probably one or more hidden layers between the operands and answers. Figure 2B demonstrates one alternative network that for present purposes is functionally equivalent to the simple network in 2A. Figure 2B shows a forward inhibition network in which excitation from the operand activates an intervening inhibitory node, which in turn depresses activity in the answer node. This forward inhibition version of the disinhibition model shows how a single excitatory source (an operand) could have both excitatory and inhibitory effects; this version is the basis of a later simulation. Other multi-node models are possible (e.g., reciprocal inhibition), and these different instantiations of the present theory might eventually lead to distinct predictions about number processing. Irrespective of the specific underlying connections and many other unresolved issues (e.g., the range of inhibited representations, variations in the strength of inhibition), the general hypothesis that inhibitory as well as excitatory functional connections develop between operands and answers has profound implications for the production of correct answers.

Disinhibition of correct answers. Given inhibitory connections between individual operands and same-table answers, it seems likely and perhaps essential that this inhibition be suppressed or overridden when the answer is correct; that is, the excitatory connection should predominate when the appropriate other operand is present. Under such circumstances, inhibition of inhibition (i.e., disinhibition) would play a major role in activating the correct answer.

Figure 2C illustrates a disinhibition mechanism of the sort hypothesized here. The presentation of 5 inhibits the inhibition from 3 to 15, which results in the dominance of the excitatory connection. Analogous connections exist between 5 and 15; that is, 5 has both excitatory and inhibitory connections to 15 and the latter is in turn inhibited by the presence of 3. The net effect of an appropriate balance of excitation and inhibition would be that 3 and 5 would excite 15 when both operands were present, and would have less impact on 15 when either operand was absent.

Rational analysis of arithmetic problems thus leads to an associative network that is isomorphic to those proposed by Williams and Clark (1992) for context-sensitive conditioning, as seen by comparing Figures 1 and 2. The notion of disinhibition thus provides an elegant explanation for the fact that activation must be highly conditional (i.e., context-dependent) in associative networks for number facts. At least in principle, disinhibitory networks could approach the ideal of 5 activating 15 only when 3 is present, 20 only when 4 is present, and so on, and could do so without positing special configural representations for problems. The plausibility of these mechanisms is further enhanced by their capacity to cope with such extreme context dependencies as ambiguous multiplication problems.

Ambiguous Problems

As noted earlier, one challenge for simple associative models of arithmetic is that multiplication, addition, and other number facts involve networks in which some pairs of operands are connected to more than one shared answer, which seems to render such problems ambiguous for models that assume all activation arises from separate operand and operator codes. For example, 4x8 is ambiguous because 4 and 8 share the incorrect answer 24, as well as the correct answer 32. There are 10 such shared-answer problems, and they result from the five multiplication answers that are correct for more than one problem (see Table 1A). Given these shared correct answers, pairs of operands can both be linked to more than one answer, albeit by different problems. Table 1B shows the 10 ambiguous problems (ignoring operand order and ties) and their incorrect answers. Note that ambiguous problems and shared-answer problems only partially overlap, having four problems in common. Problem ambiguity is even more acute in addition, where most pairs of operands are connected to more than one common answer.

Insert Table 1 about here

The ambiguous multiplication problems in Table 1 cannot be solved by simple associative models that posit only excitatory connections between representations for single operands and their answers. Summation of excitation from the individual operands (e.g., 4 and 8) fails to produce the correct response because 24 and 32 are equally activated. Because people do learn ambiguous number facts, the simple model cannot be correct, and some additional mechanism must produce greater activation of the correct response in ambiguous problems. Interestingly, the disinhibitory associative networks presented in the previous section do differentially activate correct answers for ambiguous problems.

Disinhibition and ambiguous problems. Figure 3 shows disinhibition associative networks for the 4 and 8 times-tables, including competing responses connected to both operands. Presentation of 4x8 results in combined excitation and inhibition of the answers in the 4 and 8 times tables (i.e., the answers connected to the individual operands 4 and 8). By themselves, these associations might result in minimal activation of particular answers, including 24 and 32, because the excitatory and inhibitory inputs would cancel one another, assuming comparable strengths. The critical mechanism for resolving problem ambiguity in Figure 3 is disinhibition. Specifically, the operand 4 suppresses the inhibition between 8 and 32, and the operand 8 suppresses the inhibition between 4 and 32. These mutual disinhibitory connections result in 32 being activated more than 24 when 4 and 8 occur together. The question marks for tie problems (i.e., 4x4 and 8x8) reflect doubts about whether the proposed mechanisms apply to ties (see later discussion).

Insert Figure 3 about here

To demonstrate that the disinhibition model does indeed work, a primitive simulation was prepared for the problems 4x6 and 4x8 using the two-stage model shown in Figure 2B. The program in essence computed activation levels of the answers 24 and 32 firstly as a direct function of the activation levels of the individual operands 4, 6, and 8. For this step, problem operands (e.g., 4 and 6 or 4 and 8) were simply set to a fixed 10 units of activation; operands in turn activated any answer connected to the individual operands by that amount on each cycle. These direct inputs activated 24 and 32 equally.

Each operand also produced an answer-specific negative component that depended on three factors: (a) the activation level of the primary operand (10 units as stated above), (b) a forward inhibition weighting factor (which was 0 or -.2 for the output shown here), and (c) a disinhibition weighting factor (also 0 or -.2 here) driven by the activation level of the appropriate co-operand for each answer (i.e., the operand 4 for the 8 to 32 connection and the operand 8 for the 4 to 32 connection). In essence, activation of the co-operand negated some of the inhibition from the operand to the answer. Activation of operands was simply "turned on" and additive growth in the activation of the answers 24 and 32 was measured over 15 cycles. For those interested in the details of this crude simulation, the short program used is presented in the Appendix.

The results of the simulation appear in Figure 4. The heavy solid line in Figure 4 shows the equivalent activation level achieved by the answers 24 and 32 when 4 and 8 were activated and the forward inhibition and disinhibition parameters both equalled zero (i.e., INH=0). This curve simply demonstrates the ambiguity of such problems for simple associative models.

Insert Figure 4 about here

The interesting results that demonstrate how forward inhibition and disinhibition can disambiguate such problems are the curves for INH=-.2. A small amount of forward inhibition (-.2) partly disinhibited by the other operand (-.2) separates somewhat the growth curves for 24 and 32, with the answer 32 increasing in activation faster than the answer 24 and achieving a slightly higher ultimate level of activation. An identical value (-.2) was used for both the inhibitory and disinhibitory connections for convenience, and has no particular importance. The slight amount of separation shown could be increased by manipulating inhibitory components of the disinhibition model, but modest separation was retained deliberately to emphasize the benefits of lateral inhibition presented later.

That inhibitory mechanisms can produce differences in levels of activation for ambiguous problems supports the proposed inhibition and disinhibition mechanisms. Analogous mechanisms can model other context effects in number processing, such as competition between different number operations.

<u>Inter-operation Competition</u>

The concept of disinhibition as just described can be used to build associative networks that are sensitive to the arithmetic operator accompanying the operands. The numbers 4 and 8, for example, are associatively related to 12 in the addition associative network and to 32 in the multiplication network. Which of these answers is activated should depend on the operator that occurs explicitly or implicitly with the operands (plus, +, times, x). Presence of + should activate 12 and presence of x should activate 32. The disinhibition view encourages the inverse conceptualization of this issue; that is, + should inhibit the answer 32 and x should inhibit the answer 12. Such a network is readily realized by a disinhibition model.

Figure 5 shows one simple disinhibitory network that would permit 4 to activate 12 when both 8 and + accompany 4, and to activate 32 when both 8 and x accompany 4. This particular network builds on the network shown in Figure 3. In essence, inhibition functions as an "and" gate, permitting excitation to pass only when one or more conditions are met (i.e., the appropriate operator), in addition to the presence of the primary stimuli. A similar network would exist between the operand 8 and the two answers, but with 4 now acting as the modulatory context.

Insert Figure 5 about here

The idealized model presented in Figure 5 is undoubtedly an oversimplification of actual arithmetic networks, ignoring as it does the relative strengths of the various connections and effects due to the order in which number operations are learned. Moreover, other inhibitory networks can probably be constructed to achieve this same specificity of activation. But the incompleteness and the specific details of the network are not important; primarily it demonstrates in principle that suitable inhibitory and disinhibitory associative connections can indeed produce selective activation of different number facts depending on what operands and mathematical operators are present.

Empirical Considerations

Strong tests of the proposed mechanisms are complicated by the hidden nature of the inferred structure and the existence of alternative possible networks that could effect inhibition and disinhibition of answers (e.g., Figure 2). Nonetheless, the disinhibition model is consistent with some findings and suggests specific mechanisms that could underlie various phenomena in number processing.

Ambiguous problems. One obvious place to look for support for the disinhibition hypothesis is the set of ambiguous problems, since such problems provide a notable example of context-sensitive activation. Several observations suggest that ambiguous problems are particularly difficult, although many factors, including differential practice, complicate comparisons across problem types.

Campbell and Graham (1985) reported that one of the most difficult multiplication problems in their set was 4x8, and they attributed this to the fact that both operands were connected to more than one answer; the most common error was 24, which is connected to both 4 and 8 via other problems. The results from the earlier simulation (Figure 4) show that 24 and 32 can both be highly activated in operand-driven associative networks. Another difficult problem was 3x9, again largely due to an error that is connected to both operands (i.e., 18).

The difficulty of ambiguous problems may also contribute to systematic deviations from the problem-size effect, the tendency for errors and RTs to increase with operand size and related measures. Campbell (1987c, Figure 6.1) plotted RTs for the 2 through 9 times-tables (excluding problems involving 0 or 1). The only nonmonotonic changes as operand size increased were for 5 and 7, with errors for 5 being exceptionally low (see also Campbell & Graham, 1985). One factor in these deviations may be the fact that 5 and 7 are the only operands not involved in any ambiguous problems (see Table 1 above). A regression analysis of RTs (estimated from Campbell's Figure) demonstrated that R^2 increased 12% ($p = .04$) from $R^2 = .80$ with just operand size (2 to 9) to $R^2 = .92$ including number of ambiguous problems in which operands appear (values of 3, 4, 4, 0, 5, 0, 2, and 2 for operands from 2 to 9, respectively). The advantage of unambiguous operands is difficult to interpret, however, because ambiguity is confounded with other beneficial factors (e.g., experience counting by fives, 5 problems end in 0 or 5).

A final hint that ambiguous problems are more difficult than expected given other properties of the problems comes from research on shared-answer problems. Campbell and Clark (this volume) report data showing that the shared answer problems in Table 1A are more difficult than expected given a variety of other strong correlates of problem difficulty (see also Campbell & Graham, 1985). They propose that these problems are analogous to the fan effect reported in other cognitive tasks (Pirolli & Anderson, 1985), but a related possibility is that problem ambiguity contribute to this effect; 4 of the 10 shared-answer problems are ambiguous. Indeed, the fan effect, which involves stimuli associated with varying numbers of facts, may be more similar to ambiguous problems (i.e., a problem connected to multiple answers) than to shared-answer problems (i.e., an answer connected to multiple problems).

One barrier to analyzing hypotheses about intercorrelated attributes is the limited number of standard arithmetic problems. Stronger tests of this and competing hypotheses might be possible, however, with the alphaplication task of Graham and Campbell (in press). In that task, subjects who learned arbitrary arithmetic-like associative networks among letters displayed many of the phenomena characteristic of number fact retrieval. Such analogue tasks permit experimental manipulation of ambiguity independent of other confounding variables.

<u>Cross-operation interference</u>. A second class of phenomena for which the proposed model shows strong promise is cross-operation interference. The disinhibition model hypothesizes that arithmetic operators suppress competing answers for alternative operators (Figure 5). Because activation is produced by the individual operands and operators connected in an integrated associative network, the theory therefore predicts that different mathematical operations should influence one another's performance.

One obvious implication is that cross-operation errors should be prevalent in production tasks, because successful multiplication entails suppression of addition answers and vice versa. This is indeed the case, especially when operations are mixed (Miller & Paredes, 1990). The effect of mixed operations on errors is explained by extra challenges for cross-operation suppression when to-be-suppressed answers are occasionally activated in the testing situation. The integrated nature of the proposed network is also consistent with research demonstrating that correct answers for other operations slow rejection of false verification problems; that is, such problems as 4x3=7 are more difficult to reject than problems such as 4x3=8 (Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986).

More specific predictions about cross-operation errors can be derived from the disinhibition model in conjunction with supplementary assumptions that appear reasonable. For example, the model may explain the fact that multiplication appears to interfere with addition more than addition interferes with multiplication (Miller & Paredes, 1990), at least as measured by cross-operation intrusions. The reasoning here is that most people learn addition before multiplication and other arithmetic operations. Therefore the network for addition could be acquired, at least initially, without suppression of the still-unlearned multiplication answers by the addition symbol; there is simply no need to suppress answers for interfering operations that have not yet been learned.

Multiplication, on the other hand, is acquired after addition, and initial learning requires suppression of the arithmetic answers from the outset. As multiplication is learned, or shortly thereafter, the addition network will need to be revised to include suppression of the newly learned multiplication answers. The specific effects of these distinct learning histories are not known, but it seems possible that learning inhibition initially might be easier or more efficient than trying to superimpose it later on an already developed system. Hence, more interference is observed from multiplication to addition than vice versa.

The inhibition-disinhibition model for the contextual effects of operation signs also predicts that learning a new operation will affect already learned arithmetic facts because the learner will need to develop cross-operation inhibitory connections. Newly-learned multiplication answers, for example, must be suppressed for addition to function well. Just such effects of learning multiplication on addition have been demonstrated in studies of children learning arithmetic (Miller & Paredes, 1990). Specifically, learning multiplication temporarily disrupts addition, slowing it down and increasing errors. This phenomena is consistent with the present hypothesis that inhibitory connections must be developed between the addition operands, the addition sign, and the new multiplication answers. These effects may also be related to developmental differences in inhibitory functioning discussed later.

Experimentally, such phenomena and perhaps more precise predictions, could be investigated by teaching novel arithmetic operations to the point of associative retrieval, or again by using the alphaplication task of Graham and Campbell (in press).

Another somewhat perplexing finding that might be explained by the present hypothesis is that addition is perhaps not as easy as expected relative to multiplication, which is learned at a later age and presumably receives less overall lifetime practice than addition. Differences in errors between multiplication and addition can be quite small and can even favor multiplication, and similar small or reversed effects have been reported for RT. Miller and Paredes (1990), for example, found that error rates for adults were only slightly higher for multiplication than addition (their Table 1, Mixed condition) and RTs were actually somewhat faster for multiplication (their Figure 1). One factor that may contribute to similar difficulties for addition and multiplication is the asymmetry in cross-operation interference just discussed; that is, multiplication interferes more with addition than vice versa. A second contributing factor could be the greater ambiguity of addition problems than multiplication problems. Because the 36 addition problems involve just 15 distinct answers, few addition problems involve a unique combination of operands and operands for the addition problems from 2 to 9 are connected to as many as 7 different shared problem answers (this maximum occurs for adjacent operands). Cross-operation errors and ambiguity both implicate inhibition.

Of course, problem and operation difficulty depend on many correlated variables, making it difficult to isolate the role of individual factors such as ambiguity. Addition involves closely related answers that might compete with one another more intensely than the more distributed answers for multiplication, although Campbell and Oliphant (this volume) describe a model based on compressed similarity for larger numbers. On the positive side, addition answers are lower numbers than multiplication answers, and addition is acquired before multiplication. Further complicating the investigation of such issues is the fact that people might differentially practice problems and operations that vary in initial difficulty, making comparisons with adults even more uncertain. At the very least, the disinhibition model suggests factors that must be studied if we hope to eventually understand the relative difficulty of various arithmetic operations in terms of percentage ambiguous problems, order of acquisition, and diverse other properties.

Ties and other "special" problems. A third finding that benefits from a consideration of inhibitory retrieval mechanisms is the fact that tie problems (or twins) and problems involving 0 or 1 are particularly easy, both in terms of few errors and fast RTs (e.g., Miller, Perlmutter, & Keating, 1984). Moreover, such problems show less of a problem-size effect than non-tie problems (Groen & Parkman, 1972; Miller et al., 1984). The classical explanation for such findings is that people use a special "rule" or a different "procedure" to solve problems involving ties, 0, or 1, whereas semantic retrieval mechanisms are used for non-ties. This explanation is not very adequate, at least from an associative perspective, because rules and procedures must somehow be realized by an associative network based on spreading excitation and inhibition.

The inhibition and disinhibition networks shown earlier suggest a simple mechanistic explanation for some of these findings. Specifically, retrieval of answers for these special problems may be less conditional or context-dependent than the remaining problems. In the case of zero, a simple network involving one operand (0), one sign (x), and one answer could easily be constructed. The value of the co-operand is irrelevant, permitting a simpler network than those involved with other operations to produce the correct response.

A similar analysis works for ties. The networks for ties are simpler than for non-ties because the presence of a single operand (albeit a duplicated one) is sufficient to elicit the response. That is, one might learn connections such as 2->4, 3->9, and so on qualified only by the multiplication sign and not qualified by co-operands. Non-tie problems necessarily entail conditional activation of the answer by signs and both co-operands. Hence, tie problems that involve a single operator may require a less complex associative structure than problems that involve several operators whose activation of answers depends strongly on the presence of specific co-operands.

It is less obvious that a similar associative analysis will work for ones problems because the specific response still depends on the operand paired with one. But an associative network should be possible in which the presence of 1 cues the naming of the other operand, again subject only to the presence of the appropriate operator. Moreover, ones problems benefit from the fact that the answer is directly primed by the operand with which it is identical (cf. Campbell & Clark, this volume).

This analysis suggests that comparisons of tie, 0, and 1 problems with other problems might be better served by a distinction between types of associative structures than by a distinction between retrieval and rule-governed performance. That is, the unconditional associative networks involved in ties, 0, and perhaps 1 may result in faster RTs and fewer errors because they involve less selective inhibition and disinhibition than other problems, although it remains to be determined whether such associative structures can explain the incidence of "rule-like" errors in normals (e.g., 4x0=4 or 8+1=8) and the atypical behavior of these problems in some acalculic subjects (e.g., Sokol, McCloskey, Cohen, & Aliminosa, 1991).

Table-related errors. Finally, the present model is entirely consistent with numerous standard findings on problem difficulty. One robust effect is that many errors in multiplication and other arithmetic production tasks are table-related (i.e., they come from one of the operand's times-tables), and incorrect verification problems are more difficult to reject when the presented answer belongs to the times-table for one of the operands (Campbell, 1987b). Table-related errors follow directly from models that posit individual connections between operands and answers, such as the present one. Campbell and his colleagues have developed and extensively tested this class of associative model (e.g., Campbell, 1987a; Campbell & Graham, 1985), and in general their results could be accommodated by the present associative model. I note later possible differences between the Campbell approach and the present mechanisms.

In the variant proposed here, table-related answers are directly activated because their operand-answer associations are in fact sometimes correct for one of the individual operands. This residual activation produces potential interference that requires greater dependence on the delicate balance of excitation and inhibition than unrelated answers. The inhibitory connections were originally proposed to suppress inappropriate activation resulting from excitatory connections between individual operands and answers. Any inadequacy in the inhibitory connections or lack of specificity in the disinhibition would permit activation of operand-related incorrect answers, especially if incorrect answers had been recently primed (Campbell, 1990; Campbell & Clark, 1989). Presentation of competing answers in a verification task similarly increases demands on the inhibitory system. Thus, table-related errors follow from the operand-based associative model proposed here.

Operand-based inhibition and disinhibition are clearly effective mechanisms for context-sensitive activation, and some interesting findings encourage further consideration of the hypothesis. However, alternative mechanisms are possible for conditional activation, such as exaggerated excitation and various types of configural representations.

Exaggerated-Excitation Models

Instead of suppressing an inhibitory connection, context might act by enhancing or exaggerating excitation. Thus, the presence of 4 might directly enhance the spread of excitation from 8 to 32, rather than suppress the spread of inhibition from 8 to 32. Exaggerated excitation based on the presence of the co-operand in principle could permit differential activation of the correct response. Although strong conclusions cannot be drawn about the relative merits of the exaggerated excitation versus the disinhibition hypotheses, several considerations presently favor disinhibition.

One practical problem for the exaggerated excitation hypothesis is that it is not obvious how or that exaggerated activation can be achieved in associative networks that incorporate mechanisms consistent with the known capacities of neural mechanisms. The dilemma is that simple summation of excitation does not produce exaggeration of existing activation; instead, the various sources of activation simply sum. For example, if the inhibitory connections of the inhibition-disinhibition model simulated in Figure 4 are made excitatory, identical activation levels result for the competing correct and incorrect answers. Although both 24 and 32 achieve higher levels of activation because of the additional excitation, the levels for the two answers remain identical.

The reason that summation of excitation does not work is that excitation has the same effect on the response in the absence of the other operand as in its presence. That is, contextual excitation elevates the level of activation in an associative system even if the contextual stimulus is the only stimulus present. Thus additive excitation from a context does not in fact modulate excitation from another stimulus. Disinhibition, on the other hand, has no effect unless it diminishes some existing excitatory input to the forward inhibition.

There are ways to overcome this difficulty, but they may entail relaxing such criteria as the physiological and psychological reality of the actual mechanisms used in the theory or in simulations of exaggerated excitation models. One simple way to exaggerate excitation, for example, is to literally multiply the activation levels of the operands; when either operand is missing, the product will be zero and that answer will not be activated. The challenge raised by this approach, however, is to discover a physiological or psychological mechanism (as opposed to a computational method) that can multiply activation levels without involving inhibitory mechanisms. This requirement entails a considerably more powerful "synaptic" associative process than the well known capacity of neural systems to summate excitatory and inhibitory impulses. Plain summation suffices for the disinhibition model.

Another difficulty for exaggerated excitation as a general mechanism for context-sensitive activation is that problematic phenomena have been identified in at least one area to which such models have been applied, namely Pavlovian conditioning. Specifically, repeated presentation of an excitatory CS in the absence of the US (i.e., extinction) weakens the capacity of the CS to elicit a conditioned response, whereas repeated unreinforced presentation of a feature in occasion setting produces little decrease in the feature's capacity to modulate responding to a CS (Williams & Clark, 1992). That occasion-setting properties of a feature are undiminished by nonreinforced presentations suggests that feature-based excitation is not involved in occasion setting, one form of context-sensitive activation. Inasmuch as inhibitory connections are thought not to be weakened by unreinforced exposure to a CS, this criticism does not apply to models based on inhibition.

A final point about exaggerated excitation is that such networks become unstable as multiple levels of contextual modulation are added. An excitatory model to explain the contextual effects of operation signs, for example, would require the product of three or more connections, and could achieve excessively high levels of activation. Because inhibition-based systems maintain modest levels of activation, additional levels of context can be superimposed without disturbing the integrity of the system. This criticism of exaggerated excitation is less telling than the others, however, because lateral or other forms of global inhibition, as discussed later, could be introduced to control any tendency to excessive excitation.

These considerations suggest that exaggerated excitation provides a less satisfactory and less mechanistic explanation for context effects than do inhibitory and disinhibitory mechanisms. A third class of theory used to explain context-sensitive activation consists of models that incorporate configural representations.

Configural Models for Selective Activation

By definition, configural models include composite representations in addition to representations for individual operands, operators, and answers. Campbell and Graham (1985), for example, proposed distinct problem representations that could uniquely activate the correct answer, resulting in differentiation of the correct and incorrect answers to ambiguous problems. Although operand representations for 4 and 8 would activate 24 and 32 equally, for example, a distinct 4x8 problem representation could differentially activate 32. Campbell and his colleagues (e.g., Campbell & Oliphant, this volume) now hypothesize modality-specific composite representations that include operands, operators, and answers as unified mental entities, and have demonstrated that such representations work well in their sophisticated model for number processing.

Despite this evidence for the sufficiency of configural representations, a number of conceptual grounds favor disinhibition models over certain types of configural models or suggest that the two approaches may yet prove to be complementary rather than mutually exclusive. Comparison of configural and disinhibition models benefits from separating "pure" configural models from "integrative" configural models.

Pure configural representations. The most extreme view would be that configural representations are novel, unitary problem representations that do not originate with existing representations for the "components" of the eventual whole. For example, configural representations might consist of perceptual analogues for entire problems that emerge independently of existing representations for individual operands and operators. The hypothesis that configural representations do not derive from component parts entails several conceptual difficulties.

Configural representations that are not aggregates of existing "parts" fail to explain how operands, signs, and other stimuli are able to activate the appropriate configural representations. Although it may be acceptable to propose that individual operands and signs automatically activate problems of which they are component parts, rejection of the componential view leaves the associative mechanism in need of further specification. A related difficulty is that table-related errors, which presumably arise from operand-driven activation, become somewhat paradoxical for theories that posit unitary representations not based on operands. Holistic representations that are not dissectible into components also sacrifice one ready explanation for the high correlations across operand order for both errors and RTs, which are most easily explained by the equivalence of their parts.

Given the various ways in which arithmetic problems can be presented, it is clearly unsatisfactory to simply suppose that holistic representations for problems are activated by the entire pattern of perceptual activation (rather than components such as 4, 6, x). Direct activation is somewhat plausible in certain cases (e.g., simultaneous visual presentation), but is strained for such cases as the sequential presentation of components in nonvisual, novel (e.g., "writing" problems on the skin by touch), or mixed modalities. Multi-stage arithmetic problems provide another common situation in which perceptual codes are not directly available; for example, the problem 4x2x4 would presumably activate the configural pattern 8x4 somehow. These various kinds of sequential presentation invite explanation in terms of component parts and associative mechanisms.

Given such limitations, the specification of the associative or other mechanisms by which components become connected to and activate configural representations are critical for evaluating the adequacy and completeness of configural models. Without specification of the actual mechanisms by which these processes operate, it is difficult to determine whether the configural models even differ from the disinhibition view, let alone which of the views is more correct. For example, a theory that includes unspecified perceptual processes by which problems activate configural representations may eventually require context-sensitive associative mechanisms equivalent to those proposed here.

Overall then, the operand-based disinhibition model is more complete and mechanistic than pure configural models, while still providing a solution to problem ambiguity. Although awkward for extreme configural models that posit pure holistic representations constructed from scratch, the preceding difficulties are less problematic for more moderate configural views, called here integrative.

Integrated configural representations. An alternative to the pure configural view (i.e., a singular and novel holistic representation) is that configural representations evolve by the consolidation or integration of existing discrete components into a unified configural network.

That is, problems exist initially as separate operands, operators, and answers. Experience strengthens the links among components, until collections of representations begin to act as integrated units rather than as separate associated elements. Such models incorporate associative learning processes, explicit part-whole relations, and other features that avoid most of the problems faced by pure configural representations.

Configural models such as Campbell and Oliphant's (this volume) clearly correspond more closely to this integrated associative view than to the extreme configural view just criticized. Their model includes such theoretical mechanisms as degree of unification and the strengthening of connections among the elements in the problem representation. Campbell and Oliphant assume that separate component representations become sufficiently integrated to function much as a single representation. That is, 4, x, 8, and 32 could start as discrete mental representations and subsequently become integrated into a "holistic" problem representation.

Such integrated configural models are in principle compatible with context-sensitive operand models. This potential correspondence raises the question, however, whether the associative structures that underlie such integrated networks involve simple direct connections, or the kinds of context-sensitive inhibitory and disinhibitory connections proposed here. The benefits of the special networks presented here are essentially those enumerated earlier; in particular, selective activation of other elements in the problem. Such networks can provide "configural representations" for problems without explicitly adding another level of holistic representation to the system, and can also explain transitional arithmetic performance prior to consolidation of the configural codes.

In a complementary fashion, Campbell's hypothesis that operands, operators, and answers constitute a single entity suggests possible modifications to the disinhibition mechanism. Specifically, configural networks should perhaps include answer-to-operand context-sensitive connections based on the presence of the appropriate co-operand. For example, 24 could activate 4 when 6 was present and 3 when 8 was present. Integrated associative networks in which all problem elements are linked in a selective way that depends critically on the presence of other elements should mimic configural representations, and could also permit a single network to accommodate inverse operations (e.g., multiplication and division).

Disinhibition and related mechanisms that permit the development of configural associative models for number processing could have far-reaching consequences for psychology, inasmuch as the distinction between associative and configural representations underlies several important unresolved issues. For example, configural or integrative representations have been proposed to explain phenomena in cued recall that seem to contradict standard associative theories (e.g., Marschark & Paivio, 1977) and to explain such context effects as patterning in Pavlovian conditioning (Williams & Clark, 1992).

Although operand-driven and integrated configural models can be reconciled and can even complement one another, fundamental theoretical questions still remain unanswered. One basic question concerns the existence of configural representations independent of the configural associative networks that define them. That is, once 4x8=32 becomes a configural unit, does it then exist in any form independent of the individual elements (4, x, 8, 32) that participate in the configural associative network? If not independent, then the elements in this network will be shared with other configural networks. If the configural unit does come to exist independent of the associative network, however, then some of the questions raised by pure configural models might resurface (e.g., how do problem parts activate the whole?). Since disinhibitory associative networks explain phenomena for which separate configural representations were created, the present analysis challenges advocates of separate configural representations to find definitive evidence that holistic representations separate from associatively-related component elements are indeed required to explain number-processing.

A related question asks whether there are single or multiple representations for each symbolic unit (e.g., operands, answers). In conditional associative networks of the sort proposed here, a single digit code could in principle participate in the multiple associative relations that underlie all number facts related to that operand. Activation spreading from the single digit code would be context-dependent because of appropriate inhibitory and disinhibitory connections. Without knowing full details of the implementation, such models as Campbell and Oliphant's suggest multiple representations for operands distributed throughout the system in combination with different problems (i.e., 4 in 4x6=24 seems distinct from 4 in 4x8=32). These replicates of 4 and other numbers may provide the tacit assumption that permits configural models to achieve context-sensitivity.

This primary section of the paper has examined the potential role of inhibition in the selective activation of problem answers; in the extreme case, to create differential activation where none existed. A related and similarly prevalent role for inhibitory mechanisms in number processing is to enhance existing differences in the activation levels of competing responses.

Inhibition Enhances Differences in Activation

Enhancement of differential levels of activation is important in number processing because competition and interference effects are so ubiquitous; that is, interfering responses are often sufficiently activated to compete seriously with the correct answer for dominance. This differentiation role for inhibitory mechanisms is more familiar and evident than context-sensitive activation; hence it is treated less extensively. I first document that number processing demonstrates high degrees of competition and interference among alternative response candidates, and then show, both in principle and empirically, that inhibitory mechanisms are ideally suited to contend with the intense competition that underlies such effects.

Interference Phenomena in Number Processing

A priori, extreme competition is expected in associative networks for numbers because each number is associated with multiple alternative responses (e.g., 3 is connected to all answers in the 3 times table, the 3 plus table, and so on). These overlapping connections mean that associative processes can activate multiple representations, some of which will interfere with production of the target response. This assumption of "noisy" associative mechanisms is supported by much evidence for interference and response competition effects in number processing, including phenomena discussed previously.

Interference within operations. Research has clearly demonstrated that arithmetic errors and RTs are highly susceptible to interference from related problems, as attested to by numerous papers in this volume and elsewhere. The dominant errors in multiplication come from answers to nearby table-related problems (e.g., 7x8=48), suggesting that nearby operands and/or problems in the same times-table are indirectly activated (Campbell, 1987a). Table-related answers also interfere with rejection of false verification problems (Campbell, 1987b).

The degree of interference exerted by related answers depends on their current activation levels. Removing certain problems from a test session, for example, speeds responding and decreases errors for any remaining problems that are susceptible to interference from the answers to the removed problems (Campbell, 1987a), and priming with table-related incorrect answers disrupts arithmetic performance (e.g., Campbell, 1987b, 1991). Moreover, competing answers can be activated by operands that are part of the answer (Campbell & Clark, this volume), and subjects may perform more rapidly during the first few moments of arithmetic testing, when competing answers would not yet be active, than later in the session (Chapman, 1915).

Close analysis of the relation between recent priming of an answer (e.g., by presentation of its problem) and the likelihood of that answer occurring as a subsequent error has demonstrated that moderately recent problems are a major source of error priming in multiplication (Campbell, 1991; Campbell & Clark, 1989). The assumption is that residual activation from recently presented problems promotes errors. Very recent problems, especially immediately preceding ones, are an exception to this rule and are discussed later.

Interference between operations. Arithmetic is susceptible to interference from other arithmetic operations, as noted earlier. Miller and Paredes (1990) reported several effects attributable to confusion between operations, including a notable incidence of cross-operation errors. That learning multiplication slowed addition also suggests conflict between answers for addition and multiplication. On verification tasks, subjects are particularly slow to reject false number problems if they involve the correct answer for a different operation (e.g., 3 x 4 = 7), as demonstrated by Winkelman and Schmidt (1974), Zbrodoff and Logan (1986), and others.

Current activation levels of competing cross-operation responses again contribute to the magnitude of interference effects. For example, mixing operations (e.g., testing both addition and multiplication within a session) increases the probability of cross-operation errors and slows responding (Miller & Paredes, 1990). Active responses presumably interfere more with the correct answer than do dormant responses.

<u>Other number-processing phenomena</u>. Non-arithmetic number-processing tasks also show interference phenomena that are relevant to calculation tasks. Consistent with the idea that numbers indirectly activate numerically close numbers, den Heyer and Briand (1986) found that subjects were faster to identify digits as digits (versus letters) when targets were preceded by a close digit than by a distant digit. Numerical nearness as a fundamental basis for associative relations among numbers is also consistent with number similarity judgments (Shepard, Kilpatric, & Cunningham, 1975) and digit naming errors (Campbell & Clark, 1989). Superfluous activation of nearby numbers could increase interference from close table-related answers, contributing to some of the interference phenomena observed in arithmetic. For example, if 7 primes 6, then 7x8 could experience increased competition from 48 because of indirect activation of 6x8.

Although other evidence for high levels of interference in arithmetic tasks could be cited (see Campbell, 1987c, and other papers in this volume), these examples demonstrate clearly that associative interference both within and between specific arithmetic operations is widespread. The errors result from situational factors and associative connections between operands and either other operands or related answers. This associative interference may be automatic or at least difficult to control, inasmuch as it is evoked even in tasks that are impaired by it (e.g., LeFevre, Bisanz, & Mrkonjic, 1988).

The preceding findings demonstrate convincingly that number processing involves activation that spreads to related representations. Elevated activation of competing answers is problematic for several related reasons. First, heightened activation of multiple responses means that such additional processes as thresholds, response criteria, and comparators are necessary to "select" the appropriate response, whereas an associative mechanism that resulted in a single response being highly activated could be said to have actually performed the selection. Second, similarly activated answers make it very difficult or even impossible to set a response criterion that is relatively error-free. Strongly differentiated responses therefore permit simpler and perhaps more mechanistic models of arithmetic.

Observed interference phenomena presumably represent residual effects that elude whatever control mechanisms exist to suppress competing answers. Given the high degree of overlap in associative networks for numbers and evidence for pronounced interference in various number processing dysfunctions (see later discussion), it seems reasonable to assume that number processing generally involves quite intense competition among correct and incorrect answers. A number-processing system that is so prone to interference would clearly benefit from some mechanism to reduce the competition; inhibition provides just such a mechanism.

Inhibitory Mechanisms and Interference

What is needed to reduce excessive interference and competition is an associative mechanism to exaggerate existing differences in activation levels of competing representations, and ideally to actually suppress the weaker candidates. Lateral inhibition among answers achieves just these results. The basic mechanism is quite simple but powerful; in essence, answers mutually inhibit activation of competing answers in proportion to their own levels of activation. The conclusion that interference might be reduced by such inhibitory mechanisms is supported by findings with analogous cognitive tasks, as well as by some simulation and empirical results in number processing.

Inhibition and analogous tasks. Indirect support for the role of lateral inhibition in number processing comes from research on analogous interference effects in diverse areas of psychology. Inhibition enjoys widespread use to suppress just such noise as underlies number processing, with selective attention providing a prototypical example of the use of inhibition to control interference.

In selective attention tasks, people attend to targets and ignore irrelevant stimuli or distracters. There is now much evidence that selective attention operates in part by suppression of distracters (e.g., Tipper, 1985; Walley & Weiden, 1973). For example, naming a color in the Stroop task is slower if the same name was a distracter on the preceding trial (i.e., had to be suppressed) than if a different color name was the preceding trial distracter (e.g., Dalrymple-Alford & Budayr, 1966). Negative priming and related suppression effects have now been observed in various attention tasks.

Selective attention provides an ideal model for the kinds of semantic retrieval operations involved in mental arithmetic. Whereas targets and irrelevant distracters are presented externally in selective attention, they are generated by internal associative processes in number processing. That is, arithmetic problems indirectly activate both "targets" (i.e., correct answers) and "distracters" (i.e., incorrect answers) to varying degrees. The resulting competition and interference in number processing are isomorphic to those found in selective attention. The parallel is even closer when distracting incorrect answers are presented directly, as in priming or verification studies. The analogy with selective attention suggests that inhibitory mechanisms could filter out or suppress competing representations in number processing, just as inhibition operates in selective attention.

This inference is further strengthened by the fact that inhibitory mechanisms suppress interfering stimuli and responses in many other domains besides selective attention and number processing. Inhibition sharpens differences in activation levels of sensory neurons so as to enhance edges (e.g., Von Bekesy, 1967), and has been proposed in the motor skills literature to resolve competition between alternative responses (e.g., Kornblum, 1965). Evidence also suggests that inhibitory mechanisms suppress competing responses in word identification (e.g., McClelland & Rumelhart, 1981) and in retrieval from semantic memory (e.g., McLeod & Walley, 1989), and contribute to such effects as retrieval inhibition in episodic memory (Roediger, 1974).

In short, the problems of competition, interference, and noise are common ones in diverse areas of psychology, and inhibitory mechanisms have been widely proposed and accepted as an effective way to reduce such competition. It therefore seems likely that inhibitory mechanisms would be similarly used to manage the excessive activation of inappropriate codes that is so widespread in the domain of number processing.

<u>Inhibition and simulations of number processing</u>. Lateral or mutual inhibition plays a major role in at least one model of number processing. Campbell and Oliphant's (this volume) model incorporates lateral inhibition among problem representations both within and between arithmetic operations. The sophistication of the model, however, makes it difficult to appreciate the benefits of its individual components, such as the lateral inhibition of interest here. The earlier simulation of ambiguous problems (reproduced in Figure 6) permits a clear demonstration of the positive consequences of lateral inhibition in number processing, because the parameters chosen for the disinhibition model achieved only very slight separation of the competing answers; a mere 12 units (5%) of activation separated the correct and incorrect answers after 15 cycles. This tight competition provides a challenging test case to evaluate the capacity of additional inhibitory mechanisms to separate the curves more sharply and to produce some levelling off or even decline in the activation of the incorrect answer.

Insert Figure 6 about here

The three curves discussed earlier were produced with the lateral inhibition parameter set to 0 (LI=0); that is, the answers of 24 and 32 did not inhibit one another. For the new pair of curves added to Figure 6 (INH=-.2, LI=-.2), lateral inhibition was set to -.2. That is, at each cycle the activation level of 24 was decreased by .2 times the activation level of 32 and the activation level of 32 was decreased by .2 times the activation level of 24. As shown in Figure 6, this modest lateral inhibition amplifies the difference between the activation levels of 24 and 32, and even produces a levelling off and an eventual decrease in the level of activation of the incorrect answer 24. These are exactly the effects that would facilitate differentiation of correct and incorrect answers. That the activation level of 24 is actually decreasing demonstrates how associative models with lateral inhibition can "select" answers from competing alternatives.

The simulation shows that lateral inhibition can in principle resolve competition in number processing. Moreover, the pattern of growth and separation in activation provides an elegant explanation for errors that occur when subjects are forced to respond quickly. The lower the cycle at which the response is emitted, the greater the activation of competing responses, and/or the less their separation from the eventual correct response. Increases in errors would also be expected if the inhibitory processes were somehow weakened (see later discussion of correlates). This in principle demonstration that inhibition can reduce competition among competing answers is consistent with several findings in number processing.

<u>Inhibition and number processing phenomena</u>. One clear example of inhibition in number processing comes from the research on error priming described previously (Campbell, 1991; Campbell & Clark, 1989). Although recently presented answers tend to intrude as errors in multiplication and other arithmetic tasks, the answer to the immediately preceding problem is significantly <u>less</u> likely than chance to occur as an intrusion, and intrusions primarily come from several trials back. Campbell and Clark (1989) modelled this finding by the combined effects of differentially decaying excitatory and inhibitory influences on the preceding answer. The

activation of the preceding answer was temporarily masked by rapidly decaying inhibition, so the answer is suppressed but then intrudes after several trials. Campbell (1991) went on to show that the inferred suppression of potentially interfering responses from the preceding trial actually produces fast RTs for immediately following related problems, in theory because one potent competitor has already been inhibited.

Interference effects themselves can be viewed as support for the notion of inhibitory mechanisms, inasmuch as interfering answers must exert their influence on targets somehow and one plausible mechanism is inhibition. The negative effects of interfering problems in a testing set, for example, may occur because active competing responses inhibit target responses more than do inactive competing responses, or because the target response must suppress competing responses, which takes longer and is more difficult when competing responses are highly activated. Either process requires inhibitory mechanisms.

Several findings on acalculia also implicate inhibitory mechanisms. Arithmetic errors in acalculics are similar to those made by normals under speeded conditions (e.g., Campbell & Clark, this volume; Sokol et al., 1991). One explanation for such findings is that brain insults selectively damage inhibitory mechanisms, which leaves noisy activation that is less constrained than normal. The selective vulnerability of inhibitory mechanisms is discussed later. Campbell and Clark (1988) used a similar model to explain errors on number naming by HY, a patient studied by McCloskey and his colleagues (McCloskey, Sokol, & Goodman, 1986). HY's errors were primarily number names that were visually similar, numerically close, and of the same odd-even agreement as the target. This is the expected outcome given damage to inhibitory mechanisms that normally filter out interfering responses similar to the target response.

Inhibitory mechanisms might also be reflected in the effects of response uncertainty on RT. In picture naming and other semantic retrieval tasks, RT increases as the number of alternative responses increases (Clark, 1991b; Lachman, 1973; Paivio, Clark, Digdon, & Bons, 1989). One explanation for uncertainty effects is that the multiple responses inhibit one another, slowing RT. A similar uncertainty effect occurs when the number of different stimuli (hence responses) are varied experimentally. Schvaneveldt and Staudenmayer (1970) found that such uncertainty effects were large for numerical tasks that required subjects to retrieve one or more responses from semantic memory (e.g., add 3 to the stimulus digit, subtract the stimulus digit from 9, or retrieve number words that had been randomly associated with the stimulus digits). Uncertainty effects were small and sometimes minimal for tasks less dependent on selective retrieval mechanisms (e.g., name the stimulus digit, add one, or subtract one). If uncertainty effects reflect inhibitory mechanisms, as suggested above, then these findings implicate inhibitory mechanisms in retrieval of arithmetic facts. As observed earlier, difficulties associated with ambiguous and shared-answer problems can also be conceptualized in terms of the fan effect, one type of uncertainty effect (e.g., Pirolli & Anderson, 1985).

In summary, inhibition can accentuate differences in activation of competing nodes in semantic memory. This role is consistent with the widespread use of inhibitory mechanisms for suppression of interfering events in other domains of psychology, simulations of number processing, and suggestive number processing phenomena. It is not clear that noninhibitory mechanisms could achieve similar differentiation of highly confusable nodes. Moreover, until alternative models are stated explicitly, it may even be difficult to determine which models actually entail inhibitory mechanisms and which do not. Models that raise or lower threshold levels to suppress errors, for example, might be characterized in terms of inhibitory associative mechanisms that change baseline activity levels of nodes. Reducing inhibition would lower the threshold (i.e., weaker signal required to "fire") and adding inhibition would raise the threshold (i.e., stronger signal required to "fire").

Inhibition-Related Correlates of Normal and Dysfunctional Number Processing

I have argued that inhibitory mechanisms serve at least two major functions in number processing, selective activation of answers and enhancement of differences in activation of competing answers, and that these functions underlie performance on many numerical tasks. This hypothesis not only explains much available data, but also suggests future research on number processing and its relation to diverse areas of psychology. Here I examine characteristics of people that ought to correlate with their number processing competencies, given the assumption that inhibitory mechanisms play a central role in arithmetic competencies. The basic approach is to examine human traits that can be demonstrated to reflect differential levels of inhibitory functioning. The areas examined are: direct indicators of inhibitory functioning, brain injury, aging, and childhood development.

Indicators of Inhibitory Functioning

One clear implication of the present proposals is that direct measures of inhibitory functioning ought to correlate with performance on numerical tasks. Several physiological and behavioral indicators of inhibitory functioning are available, although not enough is known at present about their correlations with number processing.

Physiological indicators. One physiological indicator of weakened inhibition is neuroleptic seizures. Much evidence is consistent with the hypothesis that seizures often result from and hence reflect inadequate physiological inhibition (Avoli, 1988). This evidence includes: analyses of seizures induced by kindling or other experimental procedures (e.g., Burnham, 1989); the use of agonists for GABA and other inhibitory transmitters in the treatment of epilepsy (e.g., Bartholini, Scatton, Zivkovic, Lloyd, Depoortere, Langer, & Morselli, 1985); neurotransmitter assays of epileptic brain tissue (Lloyd, Bossi, Morselli, Rougier, Loiseau, & Munari, 1985); and induction of seizure activity by antagonists of inhibitory neurotransmitters or other manipulations (e.g., hypoxia, pyridoxine deficiency) that selective damage inhibitory mechanisms (McCormick, 1989).

Epilepsy and seizures correlate with performance in a variety of psychological domains, including several relevant to inhibitory functioning. For example, Telzrow (1985) reviews evidence that epileptics have difficulty suppressing distracters in attention tasks, act impulsively and show other behaviors associated with hyperactivity, and show some evidence of aggressiveness (i.e., failure to suppress hostile behaviors).

With respect to numerical and related cognitive tasks, seizures tend to be associated with general cognitive dysfunctions, such as low scores on standardized intelligence tests, and with specific learning problems (Telzrow, 1985). Consistent with the hypothesized importance of inhibitory functioning in number processing, Green and Hartlage (1971; cited in Telzrow, 1985) reported particularly poor performance in arithmetic.

Various recording and imaging techniques can also be used to measure or infer levels of inhibitory functioning. Dustman and his colleagues (e.g., Dustman, Snyder, & Schlehuber, 1981) have argued that correlations between visual evoked potentials for patterned (checkerboards) and unpatterned (homogenous) stimuli reflect variations in inhibitory functioning. The argument is that lateral inhibition enhances the contrast in the patterned stimuli, making them more distinct from the unpatterned stimuli, and hence reducing the correlation between the two. I later show that aging and childhood development, two variables associated with changes in similarity of evoked potentials to patterned and unpatterned stimuli, are related as expected to changes in number processing performance. To my knowledge, direct tests of the theoretical relation between Dustman's measure and number processing are not available.

Behavioral measures. I have already mentioned several experimental tasks and phenomena that could serve as measures of inhibitory functioning; for example, susceptibility to negative priming effects in selective attention (e.g., Tipper, 1985) or to uncertainty effects. A more typical individual difference measure that implicates inhibitory functioning is impulsivity, which has been measured by ratings of normal and pathological behavior (e.g., Goyette, Conners, & Ulrich, 1978), and by fast but error-prone performance on the Matching Familiar Figures Task (Kagan, 1965).

These variables have been shown to correlate with cognitive functioning generally.

Impulsivity and attentional problems are primary symptoms of Attention Deficit Hyperactivity Disorder, which has been strongly associated with learning disabilities and other school-related dysfunctions, and the slope relating individual RTs to uncertainty predicts performance on intelligence tests (Jensen, 1987). These general effects suggest that individual differences in negative priming, uncertainty effects, and impulsivity might very well correlate with performance on number processing tasks. Whether the relations will be precise enough to isolate unique contributions of inhibition and of number processing remains to be determined.

Brain Damage and Related Conditions

The exceptional roles proposed here for inhibitory mechanisms also predict negative effects of brain insult on number processing, inasmuch as inhibitory brain mechanisms are more vulnerable to insult than are excitatory mechanisms. Early behavioral evidence suggested that inhibitory mechanisms were particularly vulnerable to hypoxia; for example, decreased oxygen at altitude produces exaggerated reflexes (e.g., van Harrevald, 1939; van Liere, 1942). Contemporary research has confirmed these early ideas. For example, inhibitory neurotransmitters such as GABA are particularly susceptible to hypoxia (Avoli, 1988), and one consequence of head injury can be seizures or abnormal brain activity (Mathieson, 1982). Brain injury also tends to produce other signs of weak inhibition, such as attentional difficulties (Posner, 1988), agitation, and impulsivity.

Selective damage to inhibitory mechanisms should produce number processing dysfunctions that resemble associative networks in which inhibition is relatively weak; hence, exaggerated interference effects are expected. As noted earlier, McCloskey et al's (1986) patient HY produced errors that could be predicted by the numerical similarity (nearness and odd-even agreement) and visual similarity of the erroneous and correct responses (Campbell & Clark, 1988). The assumption is that in normals digits activate number representations that are visually or numerically similar to themselves, but these competing representations are suppressed by inhibitory mechanisms that are deficient in HY.

It was also noted earlier that mental arithmetic performance by brain-damaged patients demonstrates a number of effects consistent with the hypothesis that brain injuries disrupt people's capacity to suppress interfering responses. In particular, patterns of errors are exaggerations of the patterns produced by normals (e.g., table-related errors, operand intrusions), consistent with the idea that excitatory associations remain intact to activate the interfering responses but the inhibitory mechanisms that normally filter out such responses have been weakened or perhaps even eliminated.

Aging

The preceding analysis suggests further that old age ought to be associated with weakened inhibition. The reasoning is that the elderly are susceptible to various pulmonary, heart, and circulatory dysfunctions and diseases that disrupt delivery of oxygen to the brain. Prolonged or abrupt deficiencies in oxygen delivery should produce inhibitory dysfunctions in the elderly that parallel those seen with traumatic brain insult. Models of aging based on weakened inhibition have been proposed by several authors (e.g., Clark, 1991a; Dustman, Emmerson, & Shearer, 1990; Hasher & Zacks, 1988). Supporting findings include increases with age in the correlation between evoked potentials for patterned and unpatterned stimuli (Dustman et al., 1990), and decreases with age in the capacity to suppress distracting stimuli in selective attention tasks (Tipper, 1991).

Given such findings, the present hypothesis predicts that number processing will deteriorate with age, and more specifically, that the deterioration should demonstrate characteristics of a number-processing system with weak inhibition. Consistent with this prediction, Campbell and Charness (1990) found that elderly subjects showed elevated levels of substitution errors (operand intrusions from earlier stages) on a complex calculation task, and seemed to be particularly challenged by the need to maintain multiple intermediate values in working memory. Such findings are readily explained by a weakened capacity to suppress interfering events.

Childhood Development

A relation between childhood and inhibitory functioning would have major consequences because of the educational relevance of number processing. Number processing abilities that require inhibition may be especially difficult for children if inhibitory mechanisms mature late.

Childhood and inhibitory functioning. Much evidence supports the hypothesis that inhibitory mechanisms increase in strength relative to excitatory mechanisms during childhood (Bjorklund & Harnishfeger, 1990; Clark & Johnson, 1990). Childhood changes that indicate strengthened inhibition include: a dramatic decrease in susceptibility to seizures from infancy to early teens (Hauser & Hesdorffer, 1990), decreased correlation between evoked potentials for patterned and nonpatterned stimuli (Dustman et al., 1990), improved capacity to suppress distracters on selective attention tasks (Tipper, Bourque, Anderson, & Brehaut, 1989), and decreases in impulsivity as measured by the Matching Familiar Figures Task and related tests (Kagan, 1965, 1966). Children also have difficulty suppressing competing responses in choice RT tasks (e.g., Jensen, 1987) and in picture naming tasks (Clark & Johnson, 1990).

Inhibitory processes are also suggested by several major developmental theories, including Piaget. The constructs of centration and egocentrism, for example, suggest a failure to suppress or inhibit attention to some immediate and compelling property of the situation, such as height in a conservation of volume task or one's own point of view in a perspective-taking task. If inhibitory processes indeed develop later during childhood than excitatory processes, then the present theory predicts that children will demonstrate forms of number processing that reflect weak inhibition. A number of observations in the domain of number processing support this hypothesis.

Number processing in childhood. The inhibitory analysis may shed light on why children use counting or other strategies to perform such arithmetic operations as addition, rather than retrieval from semantic memory (e.g., Groen & Parkman, 1972; Restle, 1970). Although elaborate theories have been developed to model the processes by which children select direct retrieval or various procedural approaches to arithmetic problems (Siegler, 1988), the fact that children persist in "counting" and other non-retrieval methods may actually reveal something fundamental about the state of children's inhibitory systems, and about the inhibitory mechanisms required to perform arithmetic by retrieval from semantic memory. Young children can certainly learn associations much earlier than they use them in arithmetic; for example, children can name thousands of pictures long before they are able to associatively retrieve the relatively few answers to basic arithmetic questions.

Perhaps children's inhibitory systems have not yet developed to the point of permitting effective use of semantic retrieval for highly interfering sets of stimuli such as number facts. Continued use of counting and non-retrieval strategies may therefore indicate that inhibitory mechanisms are still inadequate to permit direct retrieval given the ambiguity of some problems, and the fact that all operand-answer associations are context-dependent and must therefore be suppressed on most trials.

Another developmental finding illuminated by the inhibition hypothesis is that of Miller and Paredes (1990) on inter-operation competition, specifically the effects on addition of learning multiplication. Certainly learning processes are essential for mastering such competing operations, but some of the temporal trends observed by Miller and Paredes may also reflect maturation in the basic inhibitory mechanisms posited here to underlie resolution of inter-operation competition. That is, young children may have great difficulty developing the inhibitory and disinhibitory connections that permit selection of appropriate answers given cross-operation interference.

It is also obvious that weak inhibition can readily explain why young children produce more errors at arithmetic than do adults (Miller & Paredes, 1990). Children simply lack the inhibitory mechanisms essential for the differentiation and suppression of highly interfering responses. Not until children's inhibitory systems have matured can they construct the special kinds of associations required to cope with highly interfering networks, such as those involved in arithmetic.

Direct indicators of inhibitory functioning, brain damage, aging, and childhood development illustrate the kinds of predictions that derive from the inhibition model; similar cases could have been made for weak inhibition in such other areas as psychopathology and alcohol consumption. For example, numbers are often involved in the repetitive behaviors characteristic of obsessive-compulsive disorder; indeed, an early case study referred to such behaviors as "arithromania" (cited in Rapoport, 1989, p. 91). The hypothesis that inhibitory mechanisms underlie performance on computation and other number tasks thus predicts that number processing will correlate with diverse measures of inhibitory functioning and dysfunction. Moreover some available results are consistent with this far-reaching prediction, although the inhibition model suggests much additional research.

General Discussion and Conclusions

Previous sections of the paper have established the specific capacities of inhibitory mechanisms to explain number processing phenomena. In closing, I briefly examine some broader issues raised by inhibitory associative approaches to number processing and to cognition in general.

Mechanistic Explanations

Explaining phenomena in terms of excitatory and inhibitory connections ensures that explanations are mechanistic, rather than alternative verbal descriptions that sometime later must be translated into specific mechanisms. For example, "IF-THEN" and other high-level procedures do not readily suggest concrete mechanisms and may actually discourage a deep understanding of the cognitive mechanisms that permit organisms to act in conditional ways, especially if the higher-level description is wrongly perceived as an adequate explanation. The associative networks presented here provide a specific description of the disinhibitory mechanisms that effect conditional behavior.

Stated somewhat differently, the theoretical constructs of associations, excitation, and inhibition closely approximate the concrete mechanisms used in neuroscience, whereas elaborate and complex systems would be required to model the abstract, higher-level processes characteristic of some psychological theories. The correspondence between psychological and physiological levels of explanation ensures that cognitive theories are constrained to processes presently realizable in terms of known physiological mechanisms. A reciprocal benefit is that theoretical and empirical advances in the cognitive domain are more clearly relevant to neuroscience. For example, if a well-founded cognitive model holds that multiple representations are activated and mutually inhibit one another, then neuro-imaging might be directed towards the identification of early global activity followed by more focal localized activity. It is less straightforward to identify localizable brain states associated with such high-level psychological constructs or general processes as IF-THEN and comprehension.

Unified Theory

Inhibitory and associative explanations based on patterns of associations among a few kinds of representations provide unified explanations for diverse phenomena, providing specific and concrete models for each. For example, appropriately organized inhibitory connections (specifically, disinhibition) eliminated the need for configural problem representations in number processing. This same mechanism can similarly explain context effects in animal learning (Williams & Clark, 1992) and perhaps a wide variety of other behaviors strongly dependent on context.

General theoretical mechanisms and associated research ensure that the study of number processing has broad benefits for psychology, and is relevant to many scholars and important issues. Although the correlates of inhibition section focused on number processing, for example, the inhibition hypothesis actually maintains that diverse areas of psychology depend on inhibitory mechanisms of the sort described in this paper, and may therefore demonstrate similar correlations with inhibitory functioning. Even so, number processing may be particularly sensitive to variations in inhibitory functioning, perhaps because of the high levels of interference discussed earlier.

Empirically Justified Theory

A final strength is closely related to what some have perceived as a weakness of associative approaches to cognitive theory, namely their relative lack of internal or formal constraints. There are indeed few axiomatic limitations on what can be connected to what or on the strength of the connections, other than those presented by often inadequate data. Moreover, alternative networks may be functionally equivalent at the level of description currently available. We saw earlier, for example, that operand-answer inhibition might be realized in various ways depending on the number and type of intervening or hidden nodes. There is also much latitude associated with the particular weights assigned to connections, such as the forward and lateral inhibition in the model described earlier.

Although challenging, the alternative structures and lack of formal constraints actually constitute a strength of associative models inasmuch as the human brain and associated cognitive processes seem to demonstrate the same flexibility. Moreover, the lack of formal constraints necessitates meticulous and detailed observations to realize specific associative networks that are faithful to reality. Alternative approaches that begin with unrealistic formal assumptions about the structure of the number processing system are in danger of building elaborate theories on weak foundations. Weak beginnings may be a particular problem if the assumptions are never challenged empirically. For example, no real-world constraint dictates that visual representations for digits cannot be directly connected to articulatory representations for number words, so it would be imprudent to build a theory on this assumption without first obtaining extensive empirical support. Because they avoid artificial restrictions and properties, associative theories almost by definition are based on rich empirical data, which are needed to delineate specific features of the theory.

In conclusion, empirical and rational considerations indicate that inhibitory mechanisms play major roles in number processing. Inhibitory mechanisms permit associative networks that show selective activation under highly ambiguous circumstances without the need for configural representations. Inhibition can also exaggerate even slight differences in levels of activation for distinct answers, permitting the mechanistic "selection" of responses by suitably arranged associative networks. Number processing thus implicates diverse indicators of inhibitory functioning, including physiological and behavioral indicators, brain damage, aging, and childhood development. More generally, explanations that are mechanistic, that unify distinct areas of psychology, and that are empirically well-founded recommend associative models based on excitation and inhibition as a useful approach to the study of normal and dysfunctional number processing.

Acknowledgements

I thank Jamie Campbell for comments on an earlier draft of this paper and for many constructive talks about the role of inhibition in number processing and cognition. The ideas on inhibition in Pavlovian conditioning were developed in collaboration with Doug Williams. This research was supported by Natural Sciences and Engineering Research Council of Canada grant OGP0042736 to James M. Clark, Department of Psychology, University of Winnipeg, Winnipeg, Manitoba, Canada, R3B 2E9 (E-mail: CLARK@UWPG02.UWINNIPEG.CA).

References

Avoli, M. (1988). GABAergic mechanisms and epileptic discharges. In M. Avoli, T. A. Reader, R. W. Dykes, & P. Gloor (Eds.), Neurotransmitters and cortical functions: From molecules to mind (pp. 187-205). New York: Plenum.

Bartholini, G., Scatton, B., Zivkovic, B., Lloyd, K. G., Depoortere, H., Langer, S. Z., & Morselli, P. L. (1985). GABA receptor agonists as a new therapeutic class. In G. Bartholini, L. Bossi, K. G. Lloyd, & P. L. Morselli (Eds.), Epilepsy and GABA receptor agonists: Basic and therapeutic research (pp. 1-30). New York: Raven.

Bjorklund, D. F., & Harnishfeger, K. K. (1990). The resources construct in cognitive development: Diverse sources of evidence and a theory of inefficient inhibition. Developmental Review, 10, 48-71.

Burnham, W. M. (1989). The GABA hypothesis of kindling: Recent assay studies. Neuroscience & Biobehavioral Reviews, 13, 281-288.

Campbell, J. I. D. (1987a). Network interference and mental multiplication. Journal of Experimental Psychology: Learning, Memory, and Cognition, 13, 109-123.

Campbell, J. I. D. (1987b). Production, verification, and priming of multiplication facts. Memory & Cognition, 15, 349-364.

Campbell, J. I. D. (1987c). The role of associative interference in learning and retrieving arithmetic facts. In J. Sloboda and D. Rogers (Eds.) Cognitive processes in mathematics (pp. 107-122). Oxford, England: Oxford University Press.

Campbell, J. I. D. (1990). Retrieval inhibition and interference in cognitive arithmetic. Canadian Journal of Psychology, 44, 445-464.

Campbell, J. I. D. (1991). Conditions of error priming in number fact retrieval. Memory & Cognition, 19, 197-209.

Campbell, J. I. D., & Charness, N. (1990). Age-related declines in working-memory skills: Evidence from a complex calculation task. Developmental Psychology, 26, 879-888.

Campbell, J. I. D., & Clark, J. M. (1988). An encoding-complex view of cognitive number processing: Comment on McCloskey, Sokol, and Goodman (1986). Journal of Experimental Psychology: General, 117, 204-214.

Campbell, J. I. D., & Clark, J. M. (1989). The time course of error priming in number-fact retrieval: Evidence for excitatory and inhibitory mechanisms. Journal of Experimental Psychology: Learning, Memory, and Cognition, 15, 920-929.

Campbell, J. I. D., & Graham, D.J. (1985). Mental multiplication skill: Structure, process, and acquisition. Canadian Journal of Psychology, 39, 338-366.

Chapman, J. C. (1915). A study of initial spurt in the case of addition. Journal of Educational Psychology, 6, 419-426.

Clark, J. M. (1991a, June). Inhibitory mechanisms in childhood development and aging. Canadian Psychological Association, Calgary, Canada.

Clark, J. M. (1991b, November). Effects of response uncertainty on semantic retrieval. The Psychonomic Society 32nd Annual Meeting, San Francisco, California.

Clark, J. M., & Campbell, J. I. D. (1991). Integrated versus modular theories of number skills and acalculia. Brain and Cognition, 17, 204-239.

Clark, J. M., & Johnson, C. J. (1990, May). On the central role of inhibitory mechanisms in development. University of Waterloo Conference on Child Development, Waterloo, Canada.

Dalrymple-Alford, E. C., & Budayr, B. (1966). Examination of some aspects of the Stroop color-word test. Perceptual and Motor Skills, 23, 1211-1214.

den Heyer, K., & Briand, K. (1986). Priming single digit numbers: Automatic spreading activation dissipates as a function of semantic distance. American Journal of Psychology, 99, 315-340.

Dustman, R. E., Emmerson, R. Y., & Shearer, D. E. (1990). Electrophysiology and aging: Slowing, inhibition, and aerobic fitness. In M. L. Howe, M. J. Stones, & C. J. Brainerd

(Eds.), Cognitive and behavioral performance factors in atypical aging (pp. 153-180). New York: Springer-Verlag.

Dustman, R. E., Snyder, E. W., & Schlehuber, C. J. (1981). Life-span alterations in visually evoked potentials and inhibitory function. Neurobiology of Aging, 2, 187-192.

Goyette, C. H., Conners, C. K., & Ulrich, R. F. (1978). Normative data on revised Conners parent and teacher rating scales. Journal of Abnormal Child Psychology, 6, 221-236.

Graham, D. J., & Campbell, J. I. D. (in press). Network interference and number-fact retrieval: Evidence from children's alphaplication. Canadian Journal of Psychology.

Groen, G., & Parkman, J. M. (1972). A chronometric analysis of simple addition. Psychological Review, 79, 329-343.

Hasher, L., & Zacks, R. T. (1988). Working memory, comprehension, and aging: A review and a new view. In G. H. Bower (Ed.), The psychology of learning and motivation (Vol. 22, pp. 193-225). San Diego, CA: Academic Press.

Hauser, W. A., & Hesdorffer, D. C. (1990). Epilepsy: Frequency, causes, and consequences. Epilepsy Foundation of America.

Jensen, A. R. (1987). Individual differences in the Hick paradigm. In P. A. Vernon (Ed.), Speed of information-processing and intelligence (pp. 101-175). Norwood, NJ: Ablex.

Kagan, J. (1965). Matching Familiar Figures Test. Cambridge, MA: Harvard University.

Kagan, J. (1966). Reflection-impulsivity: The generality and dynamics of conceptual tempo. Journal of Abnormal Psychology, 71, 17-24.

Kornblum, S. (1965). Response competition and/or inhibition in two-choice reaction time. Psychonomic Science, 2, 55-56.

Lachman, R. (1973). Uncertainty effects on time to access the internal lexicon. Journal of Experimental Psychology, 99, 199-208.

LeFevre, J. A., Bisanz, J., & Mrkonjic, L. (1988). Cognitive arithmetic: Evidence for obligatory activation of arithmetic facts. Memory & Cognition, 16, 45-53.

Lloyd, K. G., Bossi, L., Morselli, P. L., Rougier, M., Loiseau, P., & Munari, C. (1985). Biochemical evidence for dysfunction of GABA neurons in human epilepsy. In G. Bartholini, L. Bossi, K. G. Lloyd, & P. L. Morselli (Eds.), Epilepsy and GABA receptor agonists: Basic and therapeutic research (pp. 43-51). New York: Raven.

Marschark, M., & Paivio, A. (1977). Integrative processing of concrete and abstract sentences. Journal of Verbal Learning and Verbal Behavior, 16, 217-231.

Mathieson, G. (1982). Pathology and pathophysiology: Part 1 - Pathology. In J. Laidlaw & A. Richens (Eds.), A textbook of epilepsy (2nd ed.) (pp. 437-456). Edinburgh: Churchill Livingstone.

McCormick, D. A. (1989). GABA as an inhibitory neurotransmitter in human cerebral cortex. Journal of Neurophysiology, 62, 1018-1027.

McLeod, B. E., & Walley, R. E. (1989). Early interference in a priming task with brief masked targets. Canadian Journal of Psychology, 43, 444-470.

McClelland, J. L., & Rumelhart, D. E. (1981). An interactive activation model of context effects in letter perception: Part I. An account of basic findings. Psychological Review, 88, 375-407.

McCloskey, M., Sokol, S.M., & Goodman, R.A. (1986). Cognitive processes in verbal-number production: Inferences from the performance of brain-damaged subjects. Journal of Experimental Psychology: General, 115, 307-330.

Miller, K. F., & Paredes, D. R. (1990). Starting to add worse: Effects of learning to multiply on children's addition. Cognition, 37, 213-242.

Miller, K.F., Perlmutter, M., & Keating, D. (1984). Cognitive arithmetic: Comparison of operations. Journal of Experimental Psychology: Learning, Memory, and Cognition, 10, 40-60.

Paivio, A., Clark, J. M., Digdon, N., & Bons, T. (1989). Referential processing: Reciprocity and correlates of naming and imaging. Memory & Cognition, 17, 163-174.

Pirolli, P. L., & Anderson, J. R. (1985). The role of practice in fact retrieval. Journal of Experimental Psychology: Learning, Memory, and Cognition, 11, 136-153.

Posner, M. I. (1988). Structures and functions of selective attention. In T. Boll & B. K. Bryant (Eds.), Clinical neuropsychology and brain function: Research, measurement, and practice (pp. 173-202). Washington, DC: American Psychological Association.

Rapoport, J. L. (1989). The boy who couldn't stop washing. New York: Penguin.

Restle, F. (1970). Speed of adding and comparing numbers. Journal of Experimental Psychology, 83, 274-278.

Roberts, E. (1987). What do GABA neurons really do? They make possible variability generation in relation to demand. The Journal of Mind and Behavior, 8, 591-604.

Roediger, H. L. (1974). Inhibiting effects of recall. Memory & Cognition, 2, 261-269.

Schvaneveldt, R. W., & Staudenmayer, H. (1970). Mental arithmetic and the uncertainty effect in choice reaction time. Journal of Experimental Psychology, 85, 111-117.

Shepard, R.N., Kilpatric, D.W., & Cunningham, J.P. (1975). The internal representation of numbers. Cognitive Psychology, 7, 82-138.

Siegler, R. S. (1988). Strategy choice procedures and the development of multiplication skill. Journal of Experimental Psychology: General, 117, 258-275.

Sokol, S. M., McCloskey, M., Cohen, N. J., & Aliminosa, D. (1991). Cognitive representations and processes in arithmetic: Inferences from the performance of brain-damaged patients. Journal of Experimental Psychology: Learning, Memory, and Cognition, 17, 355-376.

Telzrow, C. F. (1985). The science and speculation of rehabilitation in developmental neuropsychological disorders. In L. C. Hartlage & C. F. Telzrow (Eds.), Neuropsychological aspects of individual differences: A developmental perspective (pp. 271-307). New York: Plenum.

Tipper, S. P. (1985). The negative priming effect: Inhibitory priming by ignored objects. The Quarterly Journal of Experimental Psychology, 37, 571-590.

Tipper, S. P. (1991). Less attentional selectivity as a result of declining inhibition in older adults. Bulletin of the Psychonomic Society, 29, 45-47.

Tipper, S. P., Bourque, T. A., Anderson, S. H., & Brehaut, J. C. (1989). Mechanisms of attention: A developmental study. Journal of Experimental Child Psychology, 48, 353-378.

van Harreveld, A. (1939). The effect of asphyxia on reflex inhibition. American Journal of Physiology, 128, 13-18.

van Liere, E. J. (1942). Anoxia: Its effect on the body. Chicago: University of Chicago Press.

Von Bekesy, G. (1967). Sensory inhibition. Princeton, NJ: Princeton University Press.

Walley, R. E., & Weiden, T. D. (1973). Lateral inhibition and cognitive masking: A neuropsychological theory of attention. Psychological Review, 80, 284-302.

Williams, D., & Clark, J. M. (1992). Inhibitory associative mechanisms in Pavlovian occasion setting. Manuscript submitted for publication.

Winkelman, J.H., & Schmidt, J. (1974). Associative confusions in mental arithmetic. Journal of Experimental Psychology, 102, 734-736.

Zbrodoff, N. J., & Logan, G. D. (1986). On the autonomy of mental processes: A case study of arithmetic. Journal of Experimental Psychology: General, 115, 118-130.

Appendix

Simulation Program

```
DECLARE FUNCTION in! (o1!, o2!)
fi = -.2:        REM Op-Ans Forward Inh (-) / Exc Parameter
di = -.2:        REM Co-op Disinhibition (-)/ Exag Exc Param.
li = -.2:        REM Ans-Ans Lateral Inhibition Parameter
a4 = 10: a8 = 10:  REM Activation Levels for Operands

DEF fnmax (x, y)
 IF x >= y THEN
   fnmax = x
   ELSE : fnmax = y
   END IF: END DEF
DEF fnin (o1, o2) = o1 + fnmax(o1 + o2 * di, 0) * fi

CLS : PRINT "FI="; fi; " DI="; di; " LI="; li: PRINT
PRINT "Cy  Answ24    Answ32"

FOR c = 0 TO 15
 PRINT USING "## ###.###   ###.###"; c; a24; a32
 a24 = a24+fnin(a4, a6)+fnin(a6, a4)+fnin(a3, a8)+fnin(a8, a3)
 a32 = a32+fnin(a4, a8)+fnin(a8, a4)
 a24pre = a24
 a24 = fnmax(a24 + li * a32, 0)
 a32 = fnmax(a32 + li * a24pre, 0)
 NEXT c
```

Table 1

Shared Answer and Ambiguous Problem Sets

A. Shared Answer Problem Set

Answer	Problems	
12	2x6*	3x4*
16	2x8	4x4
18	2x9	3x6*
24	3x8	4x6*
36	4x9	6x6

B. Ambiguous Problem Set (Correct Answer Underlined)

Problem	Multiple Answers									
2x3	<u>6</u>		12		18					
2x4		<u>8</u>	12	16						
2x6*			<u>12</u>		18					
3x4*			<u>12</u>			24				
3x6*			12		<u>18</u>	24				
3x9					18		<u>27</u>			
4x6*			12			<u>24</u>			36	
4x8				16		24		<u>32</u>		
6x8					24				<u>48</u>	
6x9					18				36	<u>54</u>

* Problems that appear in both sets.

Figure Captions

Figure 1. Disinhibition models for occasion setting and patterning.

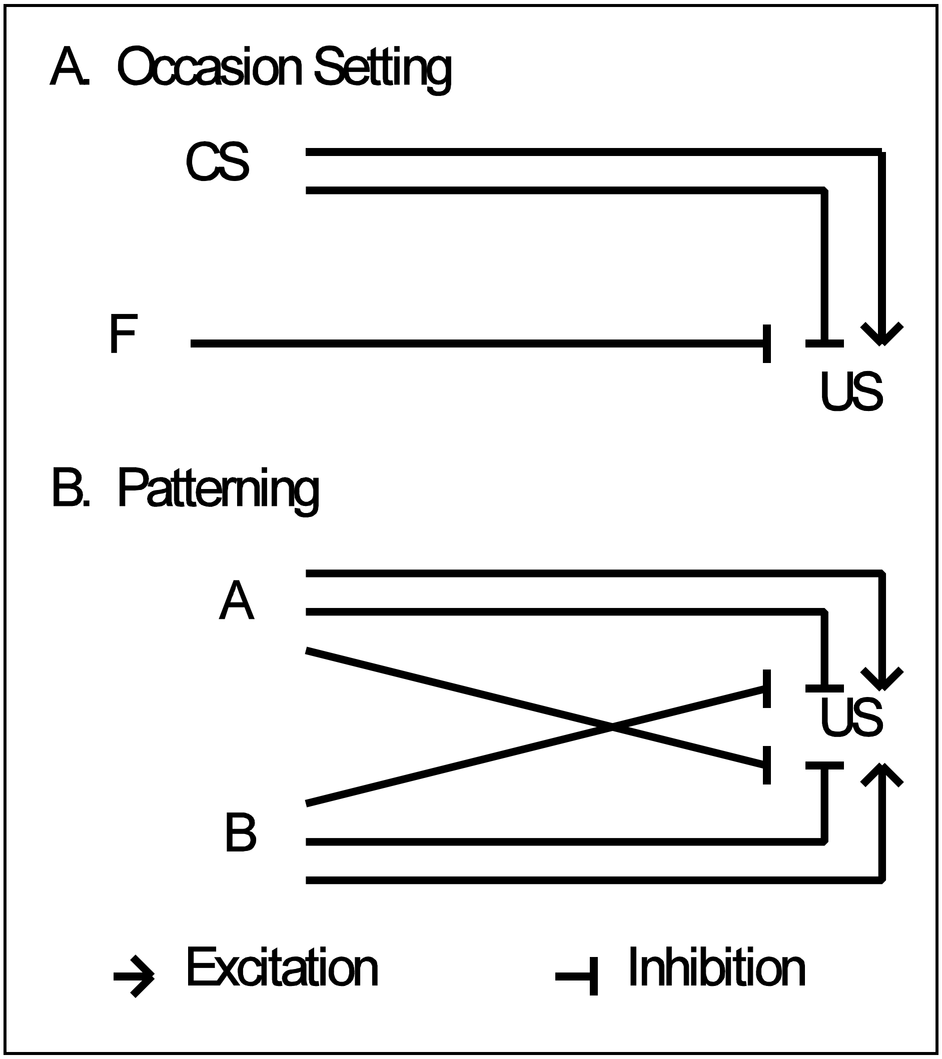
Figure 2. Inhibition-based models for multiplication.

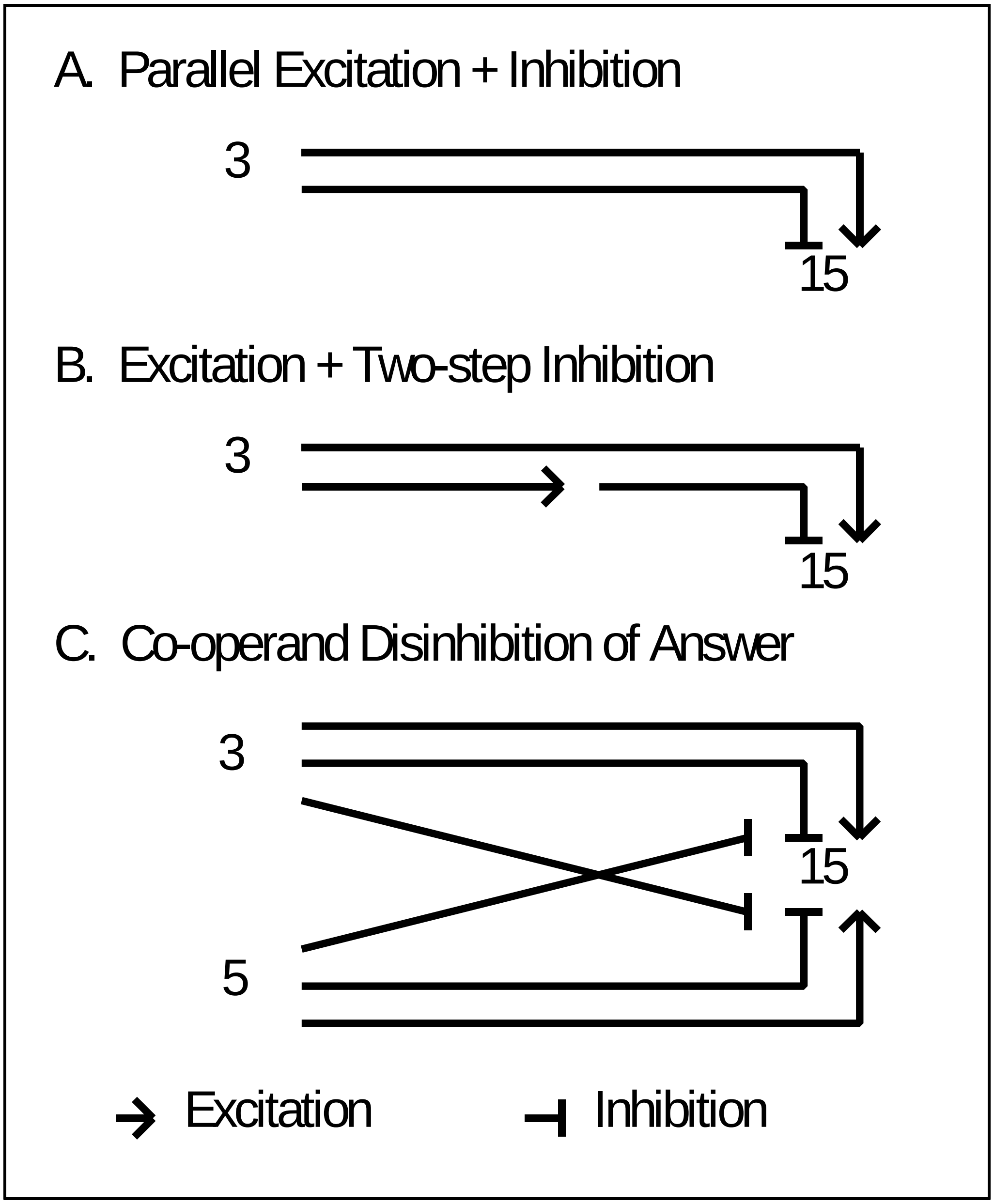
Figure 3. A disinhibition model for the ambiguous multiplication problem 4x8.

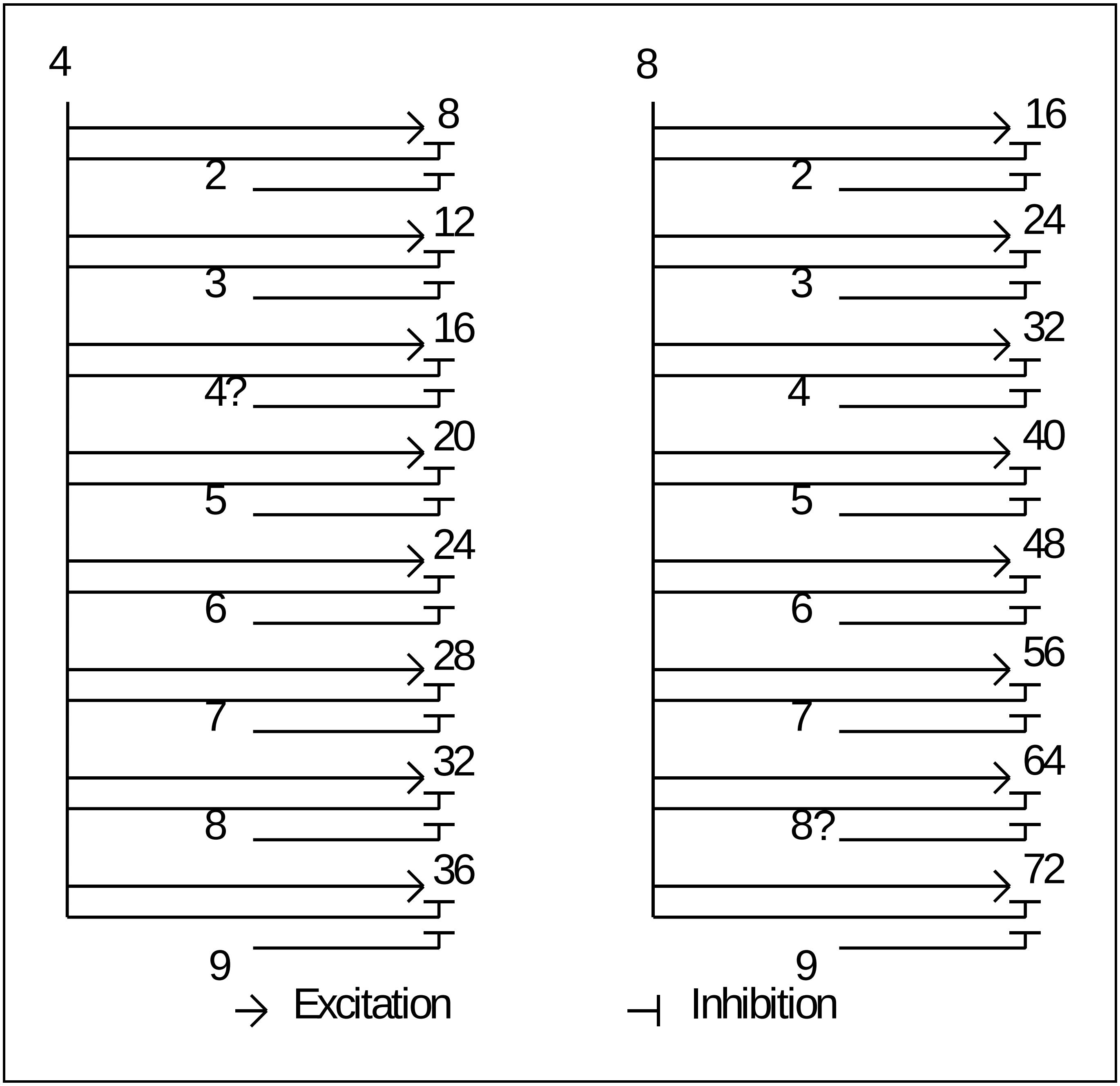
Figure 4. Simulated 4x8 activation of 32 and 24 by levels of answer inhibition and co-operand disinhibition (INH = 0 or .2).

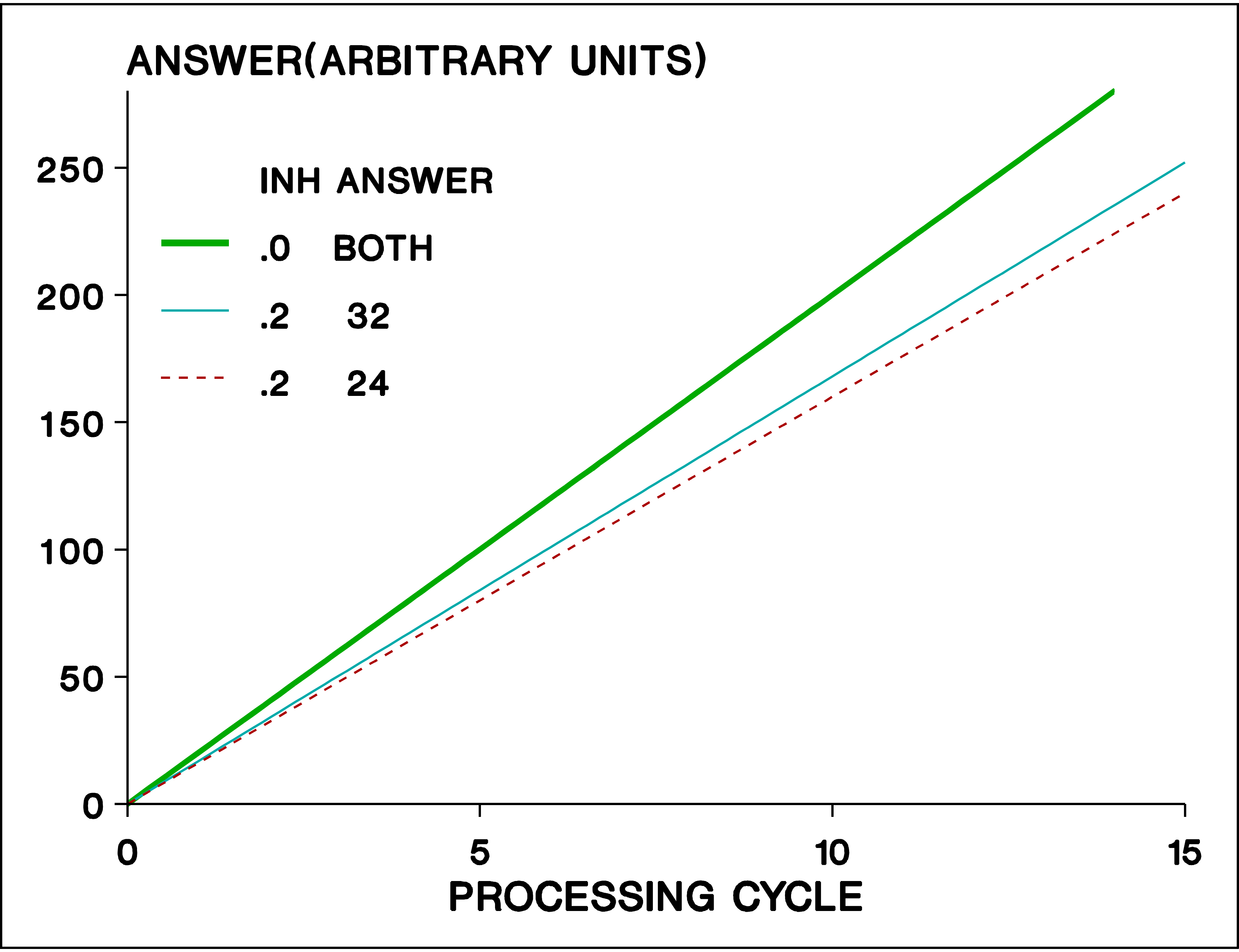
Figure 5. A disinhibition model for selective inter-operation activation.

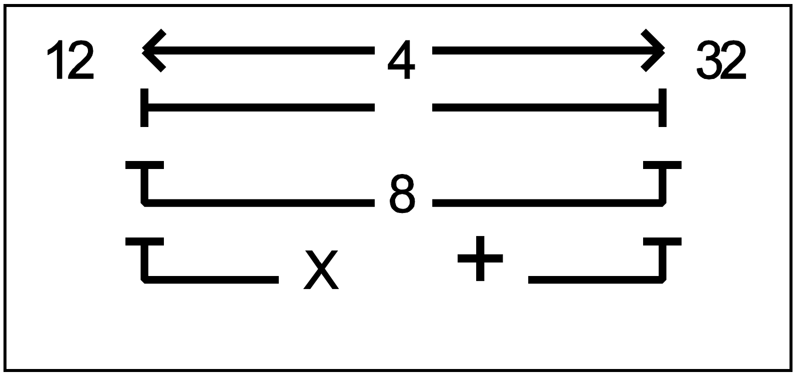
Figure 6. Simulated 4x8 activation of 32 and 24 by levels of answer inhibition and co-operand disinhibition (INH = 0 or .2), and by levels of lateral inhibition between answers (LI = 0 or .2).

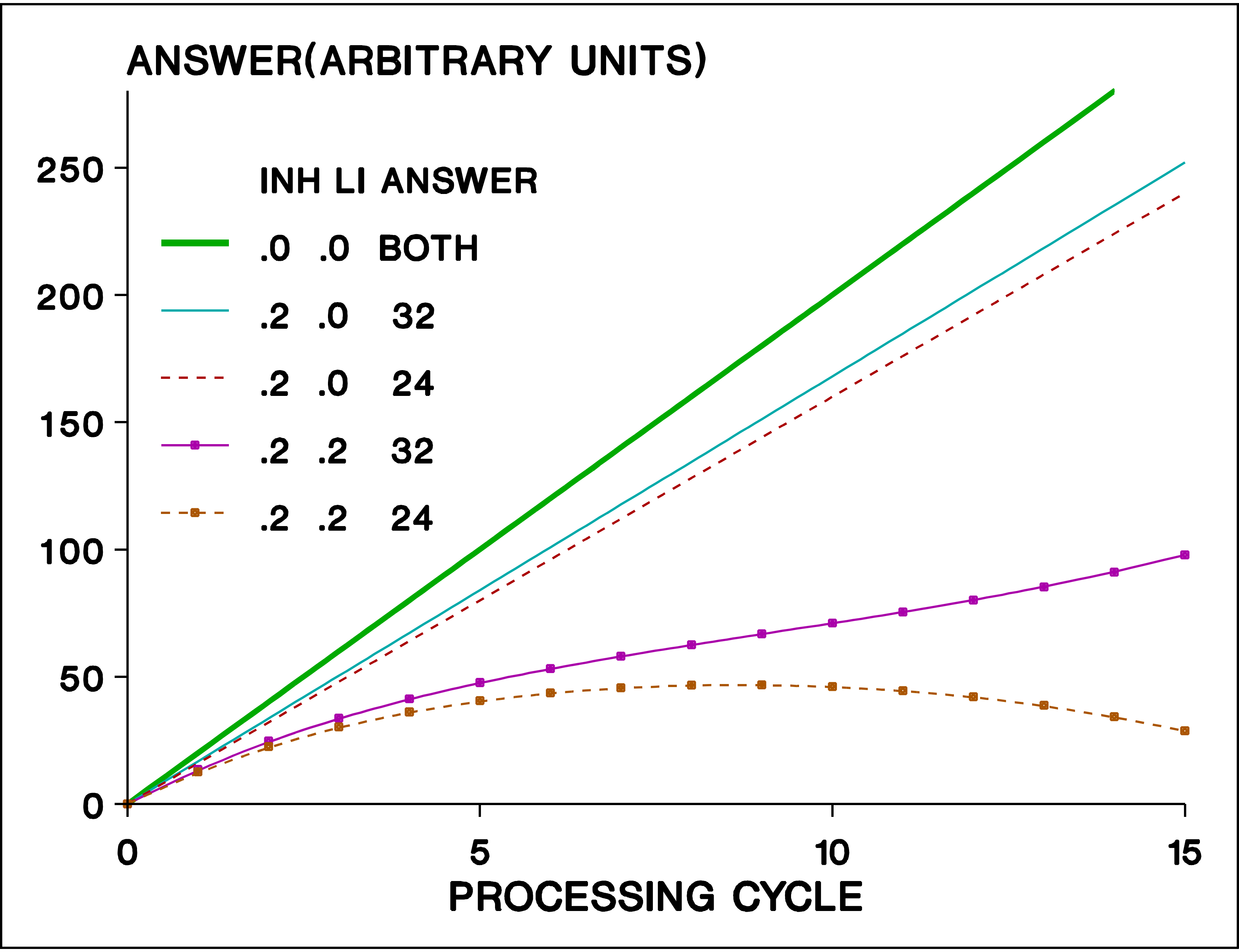

A. Occasion Setting
CS
F
US
B. Patterning
A
US
B
Excitation
Inhibition


A. Parallel Excitation + Inhibition
3
15
B. Excitation + Two-step Inhibition
3
15
C. Co-operand Disinhibition of Answer
3
15
5
Excitation
Inhibition


4
8
2
12
3
16
4?
20
5
24
6
28
7
32
8
36
9
8
16
2
24
3
32
4
40
5
48
6
56
7
64
8?
72
9
Excitation
Inhibition


ANSWER(ARBITRARY UNITS)
250
200
150
100
50
0
INH ANSWER
.0 BOTH
.2 32
.2 24
0
5
10
15
PROCESSING CYCLE


12
4
32
8
X
+


ANSWER(ARBITRARY UNITS)
INH LI ANSWER
.0 .0 BOTH
.2 .0 32
.2 .0 24
.2 .2 32
.2 .2 24
250
200
150
100
50
0
0
5
10
15
PROCESSING CYCLE